BRIDGE AND STRUCTURAL DESIGN

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PREFACE TO THE FIRST EDITION

THIS book has been developed from lectures given by the author during the past five years, under the auspices of the Dominion Bridge Co. His object has been to teach the elements of bridge and structural design in a simple and practical manner. Arts. 1 to 14 inclusive treat of the general principles of design, and are illustrated by numerous examples; while the remaining articles are examples of typical structures, in which the stresses are analyzed, the members proportioned, and the detail carefully worked out.

Both analytical and graphical methods have been employed for obtaining stresses, and the one which seemed best suited for any particular subject has been adopted. But few are given, as it was thought unnecessary to repeat information given in any of the rolling mills' hand-books.

Although the hook is intended principally for students and draughtsmen, there are parts which may be of interest to practicing bridge designers. Particular attention is here drawn to Art. 17, which treats of the design of a knee-braced mill building; and to Art. 19, which discusses the rivet spacing and web splices in plate girders, in which one-eighth of the web plate is counted on as flange area.

W. C. T.

MONTREAL, March 10, 1905.

PREFACE TO THE SECOND EDITION

THIS work, as stated in the preface to the first edition, published in 1905, was originally developed from lectures given by the author under the auspices of the Dominion Bridge Co. The aim throughout has been to set forth the elements of bridge and structural design in a simple and practical manner, so that the man unacquainted with the higher mathematics may be led to understand the principles involved in the design of simple structures.

Notwithstanding its many imperfections, the first edition met considerable favor among the class for whom it was principally intended, viz., students and draughtsmen; and as that edition has become exhausted, at the request of the publishers, the author undertook to revise it for a new edition. This revision has resulted in an entirely new book; every line has been re-written and every illustration, re-drawn; while much new material has been added, making the volume more than double the size of the first edition.

W. C. T.

MONTREAL, December, 1909.

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BRIDGE AND STRUCTURAL DESIGN

CHAPTER I

Art. 1. Definitions

Dynamics is the science which treats of laws deduced from the observed relations existing between force and matter. The science is divided into three branches: Mechanics, Hydrostatics and Pneumatics.

Mechanics is that branch of dynamics which treats of the laws governing the action of force upon solid matter.

Hydrostatics is that branch of dynamics which treats of the laws governing the action of force upon liquids.

Pneumatics is that branch of dynamics which treats of the laws governing the action of force upon gases.

Kinetics is a branch of mechanics treating of the movement of solid bodies produced by forces not in equilibrium, as in the case of machines.

Statics is also a branch of mechanics, and treats of the laws and conditions of forces acting upon solids at rest, as in the case of bridges and other structures.

Force. A force is that cause which produces the following effects: 1st. Change of motion or acceleration of a body; 2d, an opposition or balancing of other known forces; 3d, a measurable deflection or distortion in elastic bodies. The first effect relates to the study of kinetics; the second and third to the study of statics. The action of gravity, wind, steam, and animal power, are familiar examples of force. A force is clearly defined when its point of application, direction and magnitude are known.

Strain is the distortion or change in shape of a body produced by the application of equal but opposite forces, and is measured in units of length, such as inches.

Unit-Strain is the ratio $\frac{\text{alteration in length}}{\text{original length}} = \frac{d}{l}$.

Stress is the internal resistance developed in a body when strained by the application of external forces, and is measured in units of force, such as pounds or tons.

Tensile Stress or Tension occurs when the opposing forces acting upon a body tend to pull it apart, as in the tie-bars and bottom-chord of a simple span truss.

Compressive Stress or Compression occurs when the opposing forces acting upon a body tend to compress it, as in the posts and top-chord of a simple span truss.

Shearing Stress or Shear occurs when the opposing forces acting upon a body tend to cause one section to slide past another.

Bending Stress or Bending, also called *Transverse Stress*, is a combination of tension, compression, and shear, and is produced by loading a beam transversely.

Unit-Stress in bridge and structural design is the stress per square inch of section $= \frac{\text{total stress}}{\text{area of section}} = \frac{P}{A}$.

Elasticity is that property which a material possesses of returning to its original form and dimensions when the external forces causing distortion are removed.

Elastic Limit. Within certain limits every solid body is elastic, and the strain or distortion therein is directly proportional to the forces producing it, as well as to the stress resisting it. This is known as Hooke's law. But when the straining forces are increased to a certain point, the strain will increase more rapidly than these forces; and, when relieved, the body will not entirely regain its original shape, but will be found to have taken a permanent set. The maximum stress per square inch up to which the strain increases proportionately to the stress is called the *elastic limit;* and this elastic limit varies with the material and kind of stress, whether tension, compression, or shear. For mild steel, which is the material principally used for structural purposes, the elastic limit in tension is about 32,000 lbs. per sq.in.; and, in compression, about 42,000 lbs. per sq.in. After the elastic limit of a given material has been reached, additional increments of

the external forces cause rapidly increasing increments of strain up to the breaking point.

Ultimate Strength. When a body is tested to destruction in a testing machine, the maximum stress per square inch of the original cross-section, which occurs previous to rupture, is called the *ultimate strength* of the given material. For mild steel, the ultimate strength in tension is between 60,000 and 70,000 lbs. per sq.in.

Modulus of Elasticity is the ratio $\frac{\text{unit-stress}}{\text{unit-strain}}$, and varies with the material as well as with the kind of stress to which it is subjected. The modulus of elasticity in general use is that for tensile stress, known as Young's Modulus, and represented by the letter E. Then

$$E=\frac{\text{unit-stress}}{\text{unit-strain}}=\frac{P}{A}\div\frac{d}{l}=\frac{P}{A}\cdot\frac{l}{d}.$$

in which E = modulus of elasticity in lbs. per sq.in.;

P = total tensile stress in a bar of any given material, in.lbs.;
A = area of cross-section of bar in sq.ins.;
l = original length of bar in inches;
d = total strain, or stretch, in inches.

If a bar of any given material could be stretched to twice its original length (i.e., until $d=l$) without exceeding its elastic limit, then

$$E=\frac{P}{A}.$$

Thus the modulus of elasticity may also be defined as that unit-stress to which it would be necessary to subject a bar of any given material in order to double its length, provided the bar could remain perfectly elastic.

This modulus or coefficient is constant for any one material within its elastic limit, but steadily decreases beyond this point. Unit-stresses should always be well within this limit, however, and, consequently, the modulus of elasticity may be considered constant for any given material. For most materials employed in bridge and structural design, the modulus of elasticity in compression is practically the same

as for tension, and thus the two are assumed to be equal. The modulus of elasticity enables one to compute the deflections or distortions of beams or framed structures when the forces acting on them are given; or, by measuring the distortion due to the external forces, the stresses in the various parts of a structure may be determined.

The modulus of elasticity for mild steel is about 29,000,000 lbs. per sq.in.; for wrought iron, about 27,000,00 lbs. per sq.in.; for cast iron, about 15,000,000 lbs. per sq.in.; and for timber, about 1,500,000 lbs. per sq.in.

EXAMPLE 1. It is required to know the distortion or strain in a mild steel bar of 2.0 sq.ins. cross-section and 240 ins. long when subjected to a tensile stress of 32,000 lbs.

Since $E=\frac{P}{A}\cdot\frac{l}{d}$, $d=\frac{P}{A}\cdot\frac{l}{E}$. Now $\frac{P}{A}=\frac{32,000}{2}=16,000$; $l=240$; $E=29,000,000$. Then $\frac{16,000\times 240}{29,000,000}=0.132$ in., which is the the distortion (or stretch) under these conditions.

EXAMPLE 2. A cast iron column of 6.0 sq.ins. cross-section and 120 ins. long compresses when loaded 0.10 in. It is required to know what load it is carrying.

Since $E=\frac{P}{A}\cdot\frac{l}{d}$, $\frac{P}{A}=\frac{Ed}{l}$. Here $E=15,000,000$; $d=0.10$; $l=120$. Then $\frac{P}{A}=\frac{15,000,000\times 0.10}{120}=12,500$ lbs. per sq.in.; and $12,500\times 6=75,000$ lbs., which is the load on column.

EXAMPLE 3. It is observed that a timber column, 10 ins. square and 12 ft. long, is compressed 0.0576 in. under a load of 60,000 lbs. From these data it is required to determine its modulus of elasticity.

Now $\frac{P}{A}=\frac{60,000}{100}=600$ lbs. per sq.in.; $l=144$ ins.; $d=0.0576$ in. Then $E=\frac{P}{A}\cdot\frac{l}{d}=\frac{600\times 144}{0.0576}=1,500,000$ lbs. per sq.in.

Art. 2. The Composition and Resolution of Forces

The Resultant of two or more forces is that single force which will produce the same effect as the combined action of the original forces.

The Components of a force are the several forces which, by their combined action, would have the same effect as the single force.

In Fig. 1, AB and CB are drawn so as to represent both the magnitude and the direction of two forces in the same plane and acting through the same point B. Through A and C lines are drawn parallel to the forces, intersecting in the point D. Then the diagonal DB represents both the magnitude and the direction of the resultant of the two forces AB and CB. A force of equal magnitude to DB but acting in the opposite direction would balance the original forces; or, in other words, the three forces would then be in equilibrium. The figure is called the *parallelogram of forces.*

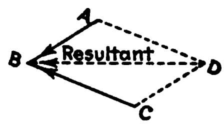


Fig. 1.

In Fig. 2 it is required to find the components of the force represented by the load P, applied at the apex of the two rafters AB and CB. A vertical line BD is drawn to represent the load P; and, from the point D, lines are drawn parallel to the rafters. Then FB, parallel to the rafter AB, is the stress in that member; and EB, the stress in CB. The horizontal lines FH and EG represent the horizontal components of the thrusts of the rafters, which are the same for both. HB is the vertical component of the thrust of rafter AB; and GB, the vertical component of the thrust of rafter CB.

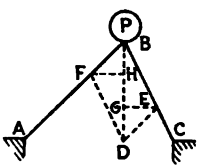


Fig. 2.

In Fig. 3 one rafter AB is inclined, and the other rafter CB is horizontal. The load P is applied at B. Then a vertical line BD is drawn to represent the load P; and, from the point D, lines are drawn parallel to the rafters AB and CB. Now FD, which is the horizontal component of the stress in AB, is equal to the stress in CB; and DB, the vertical component of the stress in AB, is equal to the load P. In other words, the rafter AB takes the whole vertical load, while CB is only subjected to a horizontal thrust. The stress in AB is equal to FB or DE.

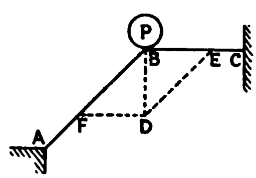


Fig. 3.

If any number of forces in the same plane meet in a point as in Fig. 4 (*a*), their resultant may be found by drawing lines end to end equal and parallel to the forces A, B, C, D, and E, as in Fig. 4 (*b*). The closing line R represents the magnitude and direction of the resultant. This diagram is called the *polygon of forces*. The arrow heads represent the directions of the forces, and follow one another around the diagram. The direction of the resultant is always towards the last force drawn. A force equal to the resultant, but acting in the opposite direction, would hold the forces in equilibrium.

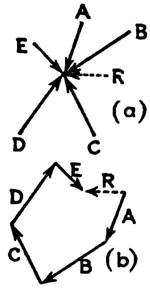


FIG. 4.

If any number of forces in the same plane and acting through the same point are in equilibrium, their force polygon will form a closed figure; and, if the direction of all the forces be known, and the magnitude of all but two, a force polygon can be constructed, from which the unknown forces may be found by scale. This principle, when applied to the forces acting at the panel-points of a bridge or roof truss, enables one to determine graphically the stresses in all of the various members.

In Fig. 5 (*a*) there are five forces acting through the same point. These forces are denoted by the letters between which they lie, which is the usual and undoubtedly the most convenient method of notation to employ in the graphical solution of problems of this nature. The forces $AB = 5{,}000$ lbs., $BC = 12{,}000$ lbs. and $CD = 25{,}000$ lbs. are given, but for the forces DE and EA the lines of action only are given. Fig. 5 (*b*) is the force-polygon, which is constructed as follows: Beginning with one of the known forces AB, the line AB is drawn parallel to and in the same direction as the corresponding force in Fig. (*a*), making its length equal to (by any convenient scale of pounds) the magnitude of the force; next, from the point B, BC is drawn parallel to, in the same direction as and of equal magnitude to its corresponding force; and, from C, CD is drawn in the same manner. The next two forces DE and EA are unknown, their lines of action only being given. Then, from the point D, a line is drawn parallel to the force DE; and, from the point A, a line is drawn parallel to the force EA. These two lines

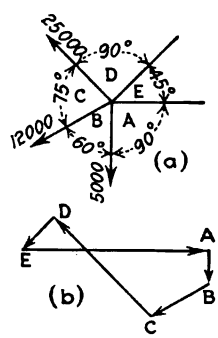


FIG. 5.

intersect in the point E, and thus the magnitude as well as the direction of the forces DE and EA are determined; for, as explained in connection with Fig. 4 (b), the arrow-heads in the force-polygon, representing the direction of the forces, follow one another around the diagram. Thus the force DE in Fig.(a) acts towards the point of intersection, and the force EA away from it. The magnitude of force DE is about 9,000 lbs. and that of EA about 35,000 lbs. Assuming that the lines of force in Fig. (a) represent the members of a framed structure, and that the forces are equal to the stresses in these members; then, when a force acts away from the panel-point, the stress in the corresponding member is tension; and, when a force acts towards the panel-point, the stress in the corresponding member is compression.

Art. 3. Examples in Graphical Statics

Example 1. Fig. 6 (a) represents a simple roof truss, the span of which is 20 ft., and its depth at the centre 5 ft. The trusses are 10 ft. apart and support (including their own weight) a load of 50 lbs. per sq.ft. of horizontal projection, concentrated at the panel-points by purlins (as in Fig. 62). The total load on one truss will then be 20 ft. × 10 ft. × 50 lbs. = 10,000 lbs.; the load at each intermediate panel-point, 10,000 ÷ 4 = 2,500 lbs.; and the load at each end panel-point, 2,500 ÷ 2 = 1,250 lbs. The loads at the end panel-point are supported directly by the walls, and do not affect the stresses in the truss.

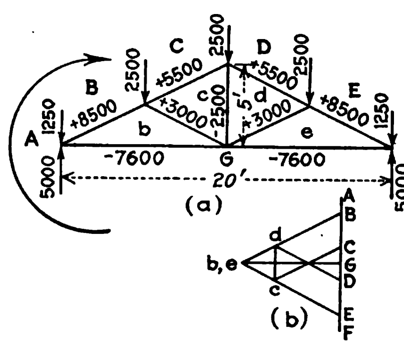


Fig. 6.

Capital letters are used to denote the external forces, consisting of the loads and the reactions of the end supports; while small letters are used in the triangular spaces between the rafters, bottom chord and web-members. The various truss members, as well as the stresses therein, are indicated by the letters between which they lie.

The stress-diagram, Fig. 6 (b), is constructed as follows: Beginning with the load AB at the left-hand end of the truss, and taking the ex-

ternal forces in regular order in going around the truss in a clock-wise direction as indicated by the curved arrow, the loads *AB*, *BC*, *CD*, *DE* and *EF* are laid off (to any convenient scale of pounds) on the vertical load line downwards. The next external force is *FG* which is the reaction at the right-hand end of truss, and equal to one-half of the total load on the span. This force is laid off upwards from *F* to *G*, as it acts in the opposite direction to the loads. Finally, the reaction *GA* at the left-hand end of the truss, which is also equal to one-half of the load on the span, is laid off upwards from *G* to *A*, the point of beginning.

Now, at the left-hand end of the truss, there are four forces meeting in a point; two of which forces, the reaction *GA* and the load *AB*, are known; and two forces, the stresses in members *Bb* and *bG*, are unknown. From the point *B* on the load line, a line is drawn parallel to the truss member *Bb*; and, from the point *G* a line is drawn parallel to the member *bG*. These two lines intersect in the point *b*, and thus determine the stresses *Bb* and *bG*. Next, at the panel-point supporting the load *BC*, the stress in *bB* and the load *BC* are known, while the stresses in *Cc* and *cb* are unknown. From the point *C* on the load line, a line is drawn parallel to the member *Cc*; and, from the point *b* in the stress-diagram, a line is drawn parallel to the member *cb*; the two lines intersecting in the point *c*, and thus determining the stresses *Cc* and *cb*. At the apex of the rafters there are now but two unknown forces, viz., the stresses in *Dd* and *dc*. From the point *D* on the load line, a line is drawn parallel to the member *Dd*; and, from the point *c* in the stress-diagram, a line is drawn parallel to the member *dc*; the two lines intersecting in the point *d*, and thus determining the stresses *Dd* and *dc*.

Since the truss and its loads are symmetrical about the centre line, it is unnecessary to proceed further with the stress-diagram, as the stresses in the right-hand end will evidently be the same as those in the left-hand end; but, in order to test the accuracy of the work, it is sometimes advisable to complete the stress-diagram for the whole truss, as has been done in Fig. 6 (*b*). When the work is carefully done, the stress-diagram will form a closed figure.

In order to know whether the stress in a member is tension or compression, it is necessary to observe the direction of the forces in the stress-diagram. In going around any panel-point in the same direction as that in which the external forces have been taken (in this case in a clock-wise direction), and taking the forces which meet in this point in reguar order, if the force in the stress-diagram acts towards

the panel-point the stress in the corresponding member is compression; and, if away from it, tension. For example: Referring to the panel-point at the apex of the rafters, and beginning with member *cC*, it will be observed that the direction of the corresponding force in the stress-diagram is towards this panel-point, which indicates compression in the member. The next force met with is the load *CD*, the direction of which in the stress-diagram is also towards the panel-point. Then comes the member *Dd* with its corresponding force in the stress-diagram acting towards the panel-point, indicating that this member is also in compression. The last member meeting in this point is *dc*, and it will be observed that the direction of the corresponding force in the stress-diagram is away from the panel-point, and so the stress in this member is tension.

The external forces, as well as the stresses in the various members, are shown on the diagram of the truss, Fig. 6 (*a*); the sign (+) indicating compression, and the sign (−) indicating tension.

EXAMPLE 2. One of the commonest forms of roof trusses is that known as the Fink Truss, so-called from the name of the inventor.

Fig. 7 (*a*) is a truss of this variety. The span is 40 ft., and the angle which the rafters make with a horizontal is 30°. The assumed concentrations at intermediate panel-points are 2,500 lbs. each, while those at the end panel-points are 1,250 lbs. each. Beginning with the load *AB* at the left-hand end of the truss, and taking the external forces in regular order, as in the previous example, the loads *A* to *J* are laid off on the vertical load line, in Fig. 7 (*b*) downwards; and the reactions *JK* and *KA* upwards, to the point of beginning.

The stress-diagram is then proceeded with, beginning at the left-hand end of the truss where there are two known forces, the reaction *KA* and the load *AB*; as well as two unknown forces, the stresses in members *Bb* and *bK*. From the point *B* on the load line, a line is drawn parallel to the member *Bb*; and, from the point *K* a line is drawn parallel to member *bK*. These two lines intersect in the point *b*, and thus determine the magnitude of the stresses *Bb* and *bK*. The former acts towards the panel-point under consideration, indicating compression; and the latter, away from it, indicating tension. At the panel-point supporting load *BC*, there are now but two unknown forces; and these are determined by drawing, from point *C* on load line, a line parallel to member *Cc*; and, from the point *b* in stress-diagram, a line parallel to member *cb*; which two lines intersect in the point *c*.

Now it will be seen that the forces Cc and cb in the stress-diagram both act towards the panel-point considered, thus indicating compression in the corresponding members. Next, at the lower extremity of member cb, there are two forces to find; so, from the point c in stress-diagram, a line is drawn parallel to member cc_1, intersecting, in the point c_1, the horizontal line passing through the point K on load line. Then the force cc_1 acts away from this point, likewise the force c_1K; thus the stresses in the corresponding members are tension. At the panel-point

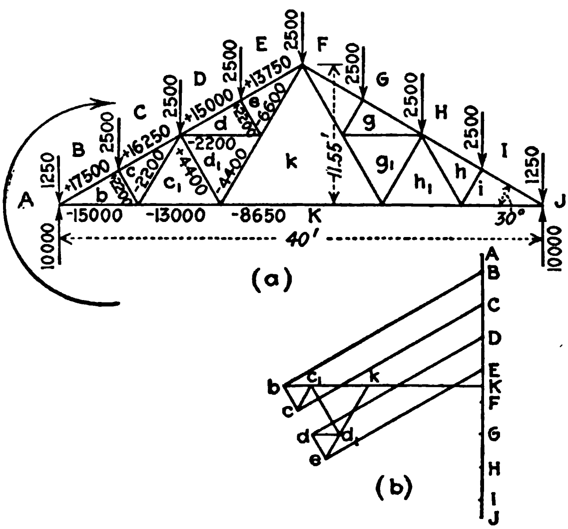


FIG. 7.

supporting the load CD, a slight difficulty is encountered; for there are three unknown forces, viz., the stresses in members Dd, dd_1, and d_1c_1. At the lower extremity of member d_1c_1 the same difficulty is met with. A nice method of solving this problem is to change some of the web-members of the truss temporarily, as in Fig. 8 (a). This arrangement leaves only two unknown forces at the panel-point supporting load CD, viz., the stresses in members Dd and dc_1, which are obtained by drawing from the point D on the load line a line parallel to member Dd; and, from the point c_1 in stress-diagram, a line parallel to member dc_1; the two lines intersecting in the point d. Now, at the panel-point supporting load DE, only the stresses in members Ee and ed are un-

known; and these are determined by drawing, from the point E on load line, a line parallel to the member Ee; and, from the point d in stress-diagram, a line parallel to member ed; the two lines intersecting in the point e. Finally, at the lower extremity of member dc_1, of the altered truss diagram, the forces meeting in this point are all known, except the stresses in members ek and kK; and these are found by drawing, from the point e in stress-diagram, a line parallel to member ek, intersecting, in the point k, the horizontal line passing through the point K on load line. The web-members may now be changed back to their

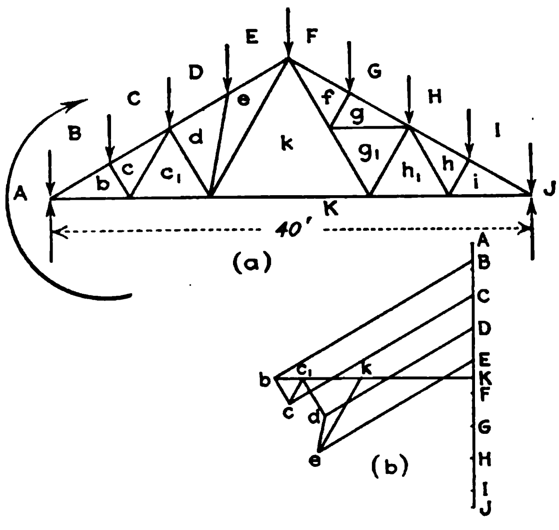


FIG. 8.

original form, Fig. 7 (a), and the polygon of forces completed for the panel-point at the lower extremity of member d_1c_1. The only unknown force at this point is the stress in member $d_1\,c_1$; and this is determined by drawing, through the point c_1, Fig. 7 (b), a line parallel to this member, intersecting the line ek (obtained from Fig. 8 (b)) in the point d_1.

When the truss is symmetrical and the loads at the panel-points equal, as in the present example, there is no difficulty in constructing the stress-diagram, for the points b, c, d, e will always lie in a straight line; but, if the panel-lengths or the loads are unequal, as is some-

times the case, it will be necessary to resort to some methods for finding an extra force, either at the upper or lower extremity of member d_1c_1.

Since the truss and its loads are symmetrical about the centre line, the stress-diagram for the left-hand half only has been constructed.

EXAMPLE 3. A common form of roof truss is shown in Fig. 9 (*a*). The span is 50 ft. c. to c. of end bearings; the depth at ends, 4 ft.

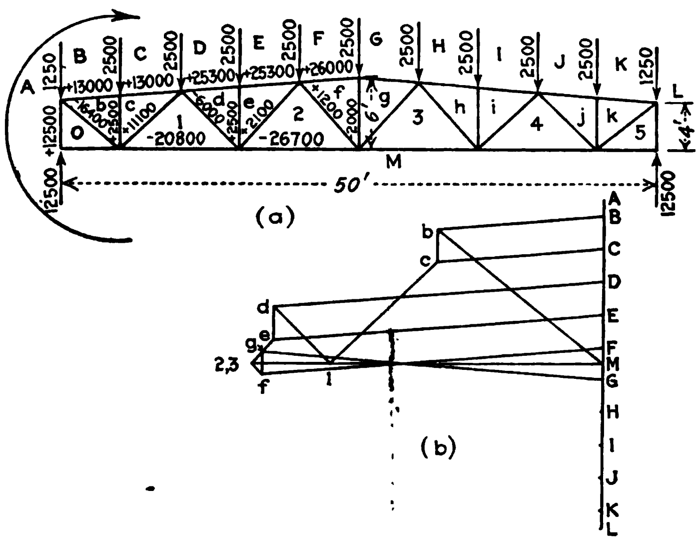


FIG. 9.

c. to c.; the depth at centre, 6 ft. c. to c. The concentrations at intermediate panel-points are assumed at 2,500 lbs. each; and at the ends, 1,250 lbs. each.

Fig. 9 (*b*) is the stress-diagram, which is only constructed for one-half of the truss. There are no difficulties to be met with as in the preceding example; for, in its development, not more than two unknown forces will be found at any panel-point. There are no stresses in the end panels of bottom chord oM and 5M due to the vertical loads: but these members are required for lateral stability.

EXAMPLE 4. Sometimes a roof truss is required to slope in one direction only, as in Fig. 10 (*a*). The span is 40 ft. c. to c. of end bear-

ings; the depth at one end, 4 ft. c. to c.; the depth at the other end, 8 ft. c. to c. The concentrations at the intermediate panel-points are assumed at 2,500 lbs. each, and at the ends 1,250 lbs. each.

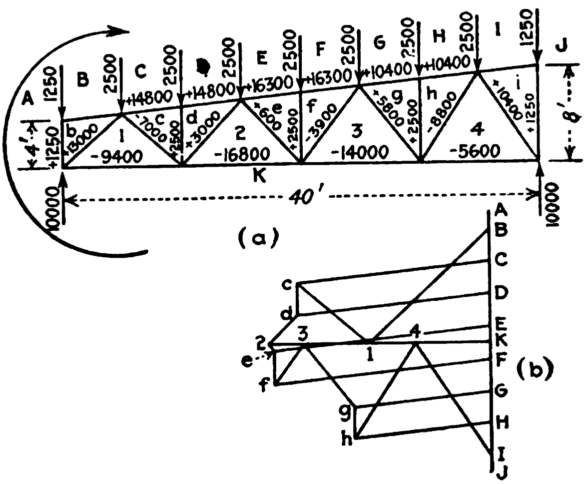
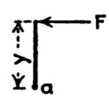


FIG. 10.

The stress-diagram, Fig. 10 (b), is constructed for the whole truss. In this case, there is no stress in the end panels of the top chord Bb and Ii due to vertical loads.

ART. 4. THE LEVER AND MOMENTS

If a force act on a body tending to rotate it about a certain point, it is said to have a moment about that point equal to the magnitude of the force (in pounds or tons) multiplied by the perpendicular distance from the line of action of force to the said point.



FIG, 11.

In Fig. 11 the force F acts about the point a with a leverage equal to y. The point a is called the point of moments; and the distance y, the lever arm. Then the moment of the force F about the point a is equal to Fy the unit being foot-pounds or inch-pounds according to the unit of length used.

There may be two or more forces tending to rotate a body about a given point, either in the same or in the opposite direction; and if

the body is in equilibrium, the sum of the left-hand (or anti-clockwise) moments must be equal to the sum of the right-hand (or clock-wise) moments. Fig. 12 represents a beam supported at the point B. The load at A tends to rotate the beam in a left-handed direction about its point of support, and its moment about this point $=3$ lbs. $\times$ 10 ft. $=30$ ft.-lbs. The load at C tends to rotate the beam in a right-handed direction, and its moment about the point $B=5$ lbs.$\times 6$ ft$=30$ ft.-lbs. Thus the moments of the loads about the point of support are equal but opposite, and the beam is balanced.

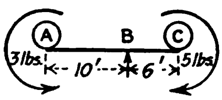


FIG. 12.

A lever may be either straight or bent; but no matter what the actual length of the lever may be, the true lever arm is the perpendicular distance from the point of moments to the line of action of the force. Fig. 13 and 14 are examples of bent levers.

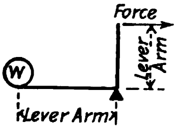


FIG. 13.

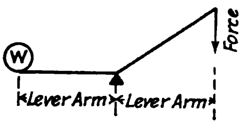


FIG. 14.

A Couple is a system of two equal parallel forces acting in opposite directions. If each of these forces be represented by P, and the perpendicular distance between them, by x, then the moment of the couple $=Px$. A couple tends to revolve the body upon which it acts, and can only be balanced by an equal couple acting in the opposite direction.

ART. 5. EXAMPLES ILLUSTRATING METHOD OF MOMENTS

The principle of the lever enters into most mechanical problems, and by its aid reactions may be determined, as well as the stresses in the various members of a framed structure. The method of moments, which is founded on the principle of the lever, will now be illustrated by a few examples.

EXAMPLE 1. In order to determine the reactions for a beam supported at both ends and loaded in any manner, the beam may be considered as a lever, pivoted at one end and supplied with an upward

force at the other end; then the sum of the moments of the loads about the pivoted end of the lever must be balanced by the moment, about the same point, of the reaction or upward force at the opposite end.

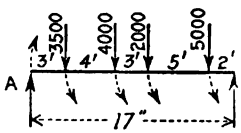


FIG. 15.

On the beam AB, Fig. 15, there are four loads, placed as shown. Assuming that the beam is pivoted at B, the sum of the moments of the loads about this point must be equal to the moment, about the same point of the reaction A. The sum of the moments of the loads about B is as follows:

$$
\begin{aligned}
5{,}000 \times 2 &= 10{,}000 \\
2{,}000 \times 7 &= 14{,}000 \\
4{,}000 \times 10 &= 40{,}000 \\
3{,}500 \times 14 &= 49{,}000 \\
\hline
& \quad 113{,}000 \text{ ft.-lbs.}
\end{aligned}
$$

Then $$\text{reaction } A \times 17 \text{ ft.} = 113{,}000 \text{ ft.-lbs.}$$

from which, $$\text{reaction } A = \frac{113{,}000}{17} = 6{,}647 \text{ lbs.}$$

To obtain the reaction at B, moments of the loads may be taken about A and dvided by the span, as before; but, since the sum of the reactions must be equal to the sum of the loads, it is only necessary to add together the loads and subtract the reaction at A, thus:

$$\text{Reaction } B = 5{,}000 + 2{,}000 + 4{,}000 + 3{,}500 - 6{,}647 = 7{,}853 \text{ lbs.}$$

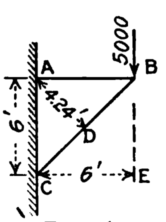


FIG. 16.

EXAMPLE 2. Fig. 16 represents a bracket on the side of a wall, supporting a load of 5,000 lbs at B. For the stress in AB, moments are taken about the point C. The moment of the load about this point $= 5{,}000$ lbs. $\times 6$ ft., while the resisting moment about the same point is equal to the stress in AB multiplied by the lever arm AC. Thus $5{,}000 \times 6 =$ stress in $AB \times 6$

$$\therefore \quad \text{stress in } AB = \frac{5{,}000 \times 6}{6} = 5{,}000 \text{ lbs.}$$

For the stress in CB moments are taken about the point A. The moment of the load about this point is the same as before; and the resisting moment is equal to the stress in CB multiplied by the lever-arm $AD = 4.24$ ft.

Then $5{,}000 \times 6 = \text{stress in } CB \times 4.24$.

$$\therefore \quad \text{stress in } CB = \frac{5{,}000 \times 6}{4.24} = 7{,}075 \text{ lbs.}$$

EXAMPLE 3. In the graphical determination of the stresses in a Fink truss, Fig. 7, Art. 3, it was found necessary to resort to a temporary change of some of the web-members in order to obtain the stress in the bottom-chord member kK. Now, if desired, the stress in this member may be readily computed analytically by taking moments, about the apex of the rafters, of all the external forces on either side of this point and dividing by the depth of the truss; for the moment, about the apex, of the stress in the bottom-chord must be equal to the moment of the external forces about the same point. The moment of the external forces about the apex is equal to the reaction KA multiplied by its horizontal distance from this point, less the loads AB, BC, CD and DE multiplied by their respective horizontal distances from the same point, thus:

$$\text{Moment at centre} = \{10{,}000 \times 20\} - \{(1{,}250 \times 20) + 2{,}500 \times (15 + 10 + 5)\} = 100{,}000 \text{ ft.-lbs.}$$

Then the stress in $kK = \dfrac{100{,}000}{11.55} = 8{,}650$ lbs., which is the same as previously determined graphically.

CHAPTER II

ART. 6. SHEARING AND BENDING STRESS IN BEAMS

Fig. 17 (*a*) represents a simple beam supported at both ends and carrying a concentrated load P. The reacton at the left-hand end of the beam due to this load P is represented by R_1, and that at the right-hand end, by R_2. Thus the load and the two reactions are the external forces which induce internal stresses, as follows:

If a section of the beam be taken between the load P and the reaction R_1 and the portion of the beam to the right of this section be removed, as shown at (*b*), then the remaining portion of the beam may be held in equilibrium by a downward vertical force Q, which is equal to the upward reaction R_1 and by two equal but opposite horizontal forces F, applied near the top and bottom of the beam. The force Q, which is termed the *vertical shear*, tends to cut the beam crosswise, and is resisted by the strength of the material in this direction. Again,

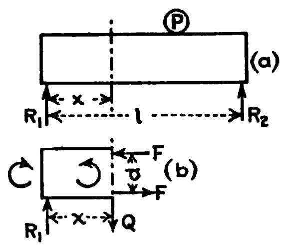


FIG. 17.

R_1 and Q form a couple, the lever-arm of which $=x$. This couple, which tends to rotate the portion of the beam shown in a clock-wise direction, is termed the *bending moment* at the section. Now the horizontal forces F also form a couple, with lever-arm $=d$. This couple tends to rotate the portion of the beam shown in an anti-clockwise direction (thus counteracting the bending moment) and is termed the *moment of resistance* of the beam at the section. For equilibrium, these two couples must be equal to one another; or

$$R_1x=Fd,$$

from which

$$F=\frac{R_1x}{d}.$$

The forces F induce compression in the upper fibres of the beam, and tension in the lower fibres.

Therefore it may be stated that a loaded beam is usually subjected, at any section, to two kinds of stress, viz., vertical shear and bending.

Shearing. The shearing stress tends to cause the particles of the beam to slide by one another in a vertical plane (as when a plate is cut in a shearing machine), and is equal to the algebraic sum of all the external vertical forces on either side of this plane. When the shearing force tends to cause the portion of the beam to the left of this plane to move upwards with respect to the portions of the beam to the right, it will be called *positive;* and, when the shearing force tends to cause the left-hand section to move downwards with respect to the right-hand section, it will be called *negative.*

Bending. The bending moment at any point of a loaded beam is equal to the algebraic sum of the moments about this point of the external forces on either side of it. This moment of the external forces must be balanced by the equal but opposite moment of the internal stresses in the beam. In the case of a lattice girder or truss, the bending moment is resisted by the moment of the stress in either chord, the lever arm being the distance c. to c. of the chords; but, in a solid beam or plate-girder, it is resisted by the entire cross-section. When the bending moment induces compression in the top chord or flange, it will be called *positive;* and when it induces tension in the top chord or flange it will be called *negative.*

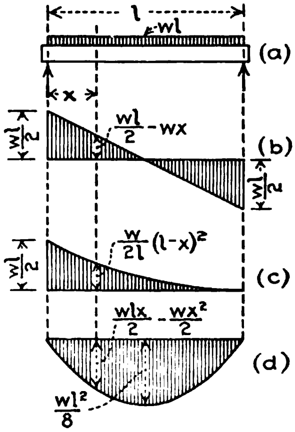


FIG. 18.

Fig. 18 (*a*) represents a beam loaded uniformly with a load w per lin. ft. At each end there are equal but opposite forces acting on the beam, viz., one-half of the total load, acting downwards, and the reaction of the support, acting upwards, each equal to $\frac{wl}{2}$. These two forces tend to shear the beam, or cut it cross-wise. The shearing force at any point distance x from the left-hand support is equal to the reaction at that support, minus the load on the length x; or,

$$\text{Shear at } x = \frac{wl}{2} - wx, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

which is the equation of a straight line, the maximum positive ordinate at the left-hand end, and the maximum negative ordinate at the right-hand end being each equal to $\frac{wl}{2}$, as shown in Fig. 18 (*b*). At the centre of the span, where the line crosses the axis, the shear is zero.

For a moving uniform load, the maximum positive shear at x occurs with the load covering all that portion of the beam to the right of this point; when the shear is equal to the left-hand reaction, which is obtained by taking moments of the load about the right-hand support and dividing by the span, thus:

$$\text{Maximum shear at } x \text{ for moving uniform load} = \frac{w}{2l}(l-x)^2, \quad . \quad (2)$$

which is the equation of a parabola with vertex at the right-hand end. When the load covers the span, $x=0$, and the maximum shear $=\frac{wl}{2}$, as before. The maximum positive shear at any point for a moving uniform load may be obtained by constructing a parabola, Fig. 18 (*c*), with vertex at the right-hand end of the span, and making the maximum ordinate at the left-hand end equal to the maximum shear, when the shear at any point will be equal to the ordinate at that point.

When there are both a fixed uniform load covering the span and a moving uniform load, the total maximum shear at any point will be equal to the sum of the two shears found independently.

The bending moment at any point distant x from the left-hand support is equal to the moment of the reaction about this point, minus the moment of the load on the length x about the same point. The reaction $=\frac{wl}{2}$, and its lever arm $=x$; the load on the length $x=wx$, and its lever arm $=\frac{x}{2}$. Then the

$$\text{Moment at } x, \text{ or } M_x = \frac{wlx}{2} - \frac{wx^2}{2}, \quad . \quad . \quad . \quad . \quad (3)$$

which is the equation of a parabola with vertex at the centre of the span. When $x=\frac{l}{2}$,

$$M_{\max.} = \frac{wl^2}{8}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Having computed the centre moment, the moment at any point may be obtained graphically by constructing a parabola, as in Fig. 18 (*d*), and making the centre ordinate equal to the maximum moment, when the moment at any point will be equal to the ordinate at that point.

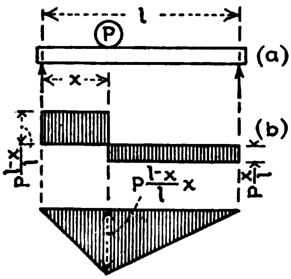


FIG. 19.

For a single load P distant x from the left-hand support of the beam, Fig. 19 (*a*), the positive shear is equal to the reaction at that end, and is obtained by taking the moment of the load about the right-hand support and dividing by the span, thus:

$$\text{Positive shear} = P\frac{l-x}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The negative shear $= P\frac{x}{l}$. The positive and negative shears are shown graphically in Fig. 19 (*b*). When $x = 0$, the positive shear $= P$, and the negative shear $= 0$. When $x = \frac{l}{2}$, the positive and negative shears are each equal to $\frac{P}{2}$.

The bending moment at the point of application of the load is equal to the left-hand reaction multiplied by its distance from this point.

$$M_x = P\frac{l-x}{l}x, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

as shown in Fig. 19 (*c*). When $x = \frac{l}{2}$, or when the load is on the centre of the span, the moment is a maximum; then

$$M_{\max.} = \frac{Pl}{4}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Fig. 20 (*a*) represents a beam supported and fixed at one end, carrying a load w per lin. ft.

The shear at any point distant x from the free end $=wx$. The shear at fixed end, or

$$\text{Shear, maximum,} = wl. \qquad (8)$$

The shears are represented graphically in Fig. 20 (*b*).

The moment at any point distant x from the free end $=\frac{wx^2}{2}$, which is the equation of a parabola with vertex at the free end of the beam, as shown in Fig. 20 (*c*). The maximum ordinate of this curve is equal to the moment at the fixed end; then

$$M_{\max.} = \frac{wl^2}{2}. \qquad (9)$$

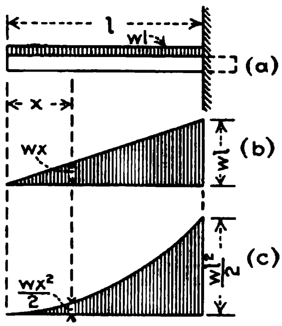


FIG. 20.

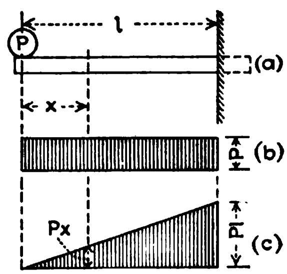


FIG. 21.

Fig. 21 (*a*) represents a beam supported and fixed at one end, carrying a concentrated load P at the outer end.

The shear, which is constant throughout and equal to P, is illustrated in Fig. 21 (*b*).

The moment at any point distant x from the load P is equal to Px, which is the equation of a straight line with origin at the free end of the beam, as shown in Fig. 21 (*c*). The maximum ordinate of this line is equal to the moment at the fixed end; or

$$M_{\max.} = Pl. \qquad (10)$$

Moment Diagram. Shears and bending moments for any system of concentrated loads may be obtained graphically by means of a moment diagram.

Fig. 22 (*a*) represents a beam supporting five concentrated loads, P_1, P_2, P_3, P_4, P_5. In Fig. 22 (*b*) these loads are laid off in regular order to a convenient scale of pounds on the vertical load line, as shown. The point *O*, called the pole, is taken at any convenient distance to the right of the load line (perferably some even number of pounds, measured by the same scale as that used for the loads on the load line); and from this pole radial lines are drawn to the points of division on the load line representing the loads. This figure is called the force diagram, and the horizontal distance of the point *O* from the load line is called the pole distance.

The equilibrium polygon, Fig. 22 (*c*), is constructed next. Beginning at any convenient point *t* on the vertical through P_1, a line is drawn to the left, parallel to the first radial line in force diagram, and intersecting the vertical through R_1 in the point *s*. Again, from the point *t*, a line is drawn to the right, parallel to the second radial line in force diagram, and intersecting the vertical through P_2 in the point *u*. From the point *u* a line is drawn to the right parallel to the third radial line in force diagram and intersecting the vertical through P_3 in the point *v*. The remainder of the equilibrium polygon *v*, *w*, *x*, *y* is constructed similarly. It should be noted that the two adjacent sides of the equilibrium polygon, to the left and right of any load, are parallel respectively to the two radial lines which pass through the upper and lower extremities of the corresponding load in the force diagram. The line *sy*, which connects the points of intersection of the verticals through R_1 and R_2 with the equilibrium polygon, is called the closing line.

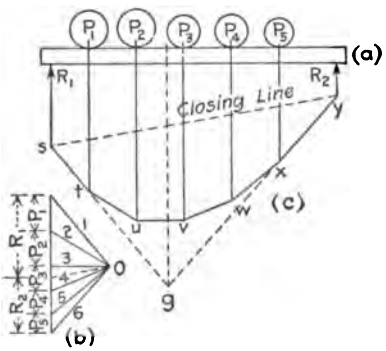


FIG. 22.

W

Now, *the ordinate at any point between the closing line and the equilibrium polygon is proportional to the moment at that point;* and this ordinate multiplied by the pole distance is equal to the moment.

A line drawn from the point O in force diagram, parallel to the closing line sy, and intersecting the vertical load line, determines the reactions. The upper part represents the left-hand reaction R_1 and the lower part represents the right-hand reaction R_2, as shown. The shear at any point of the beam between R_1 and P_1 is equal to the reaction R_1. The

shear between P_1 and P_2 is equal to the reaction R_1, minus the load P_1, and the shear between P_2 and P_3 is equal to the reaction R_1 minus the loads P_1 and P_2.

If the two outer sides of the equilibrium polygon st and xy be produced to an intersection in the point g, and a vertical drawn through this point, the vertical will represent the position of the resultant of the loads contained between the two sides produced. In other words, the vertical will determine the centre of gravity of these loads.

The moment diagram is particularly suitable for obtaining moments and shears due to wheel loads used in designing railway bridges, as more particularly explained in another book by the author entitled "The Design of Typical Steel Railway Bridges."

Art. 7. Moment of Resistance

The moment of resistance of a beam must be equal to the moment of the external forces acting upon it. When a beam is loaded transversely, it bends about the neutral axis which lies in the centre of gravity of its seection. The fibres on one side of this neutral axis are compressed and those on the other side are extended, while the fibres in the neutral axis are neither compressed nor extended. The fibres which are farthest from the neutral axis are stressed most; and, provided the maximum stress does not exceed the elastic limit of the material, the intermediate fibres are stressed in direct proportion to their distance from the neutral axis. Then the moment of resistance of a beam is equal to the moment of all the fibre stresses about the neutral axis.

The moment of resistance of a beam of rectangular cross-section may be deduced as follows:

In Fig. 23, h = depth of beam in inches,

b = breadth of beam in inches,

f_1 = max. fibre-stress in lbs. per sq.in.

The shaded portions (in elevation) represent the total fibre-stresses, which vary in intensity from zero at the neutral axis to a maximum at the top and bottom of the beam. The total stress on either side of the neutral axis is equal to the area of the triangle (whose base is equal to

the maximum fibre stress and whose altitude is equal to one-half the depth of the beam) multiplied by the breadth of the beam; and the sum of these total stresses (the one tension, and the other compression)

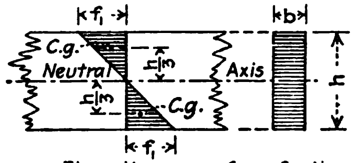


FIG. 23.

multiplied by the distance from the neutral axis to the centre of gravity of the triangles is the moment of resistance, thus:

The total stress on either side of the neutral axis $=\frac{f_1}{2}\cdot\frac{h}{2}\cdot b$; the distance from the neutral axis to the centre of gravity of the triangles $=\frac{h}{3}$. Then

$$\text{Moment of resistance of rectangle in in.-lbs.} = 2\left(\frac{f_1}{2}\cdot\frac{h}{2}\cdot b\right)\frac{h}{3} = f_1\frac{bh^2}{6}. \quad (1)$$

The factor $\frac{bh^2}{6}$, which is the same for all rectangles, is called the *section modulus*, and is represented by S. Now, if the moment of the external forces be represented by M, then

$$M = f_1 S; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$f_1 = \frac{M}{S}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$S = \frac{M}{f_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The following is a general method for determining the moment of resistance of a beam of any section.

In Fig. 24 f_1 = stress in lbs. per sq.in. on extreme outer fibres;
y_1 = distance in inches from neutral axis (through the centre of gravity of section) to the outer fibres where the stress $= f_1$;
y = distance in inches from neutral axis to any fibre;
a = an infinitely small area.

Then $\dfrac{f_1}{y_1}$ = stress in lbs. per sq.in. on fibres, at a distance of 1 in. from neutral axis;

$\dfrac{f_1}{y_1}y$ = stress in lbs. per sq.in. on fibres at a distance of y from neutral axis;

$\dfrac{f_1}{y_1}ya$ = stress on an element of fibres at a distance of y from neutral axis;

$\left(\dfrac{f_1}{y_1}ya\right)y$ = moment of stress in in.-lbs. on an element of fibres at a distance of y from neutral axis.

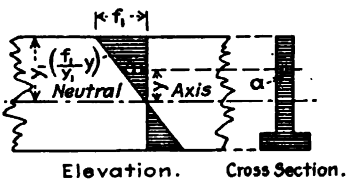


FIG. 24.

This last expression, taken for all values of y, both above and below the neutral axis, is the moment of stress (or *moment of resistance*) of the given section; or

$$M = \Sigma\frac{f_1}{y_1}y^2a = \frac{f_1}{y_1}\Sigma y^2a.$$

The factor Σy^2a is called the *moment of inertia* of the section, and is represented by I. It is obtained by multiplying each elementary area by the square of its distance from the neutral axis, and taking the sum of the products.

If the symbol I be substituted for Σy^2a, then

$$M = f_1\frac{I}{y_1}, \qquad (5)$$

$$f_1 = \frac{My_1}{I}. \qquad (6)$$

The factor $\dfrac{I}{y_1}$ is the section modulus S. For an unsymmetrical section as here shown, the section moduli for the top and bottom flanges are different. The section modulus for the bottom flange is equal to

the moment of inertia divided by the distance from the neutral axis to the outer fibres of that flange.

Calling the fibre-stress per sq.in. at a distance of y from the neutral axis f, then

$$f=\frac{My}{I}. \qquad (7)$$

It will now be necessary to consider the method of computing the moment of inertia for various sections.

Art. 8. Moment of Inertia

It has been shown in the previous article that the moment of inertia of a beam, which is represented by I, is a factor of its moment of resistance; and is equal to the sum of the products of each elementary area of cross-section multiplied by the square of its distance from the neutral axis through the centre of gravity.

For the moment of inertia of a rectangle about an axis through its centre of gravity, the section may be supposed to be divided into thin strips, as shown in Fig. 25. The length of each strip$=b$, its thickness$=t$, and the distance of its centre of gravity from the neutral axis$=y$. Then the summation of bty^2 is the approximate moment of inertia of the section; and the thinner the strips the more accurate will be the result.

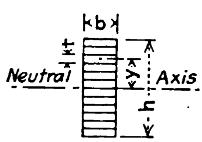


Fig. 25.

By the help of the calculus in which the thickness of each strip is made infinitely small, an exact result may be obtained, which is

$$I=\frac{bh^3}{12}. \qquad (1)$$

For this section, the moment of inertia may also be deduced from the section modulus, thus:

From the previous article $\frac{I}{y_1}=S$. Then, since $y_1=\frac{h}{2}$ and $S=\frac{bh^2}{6}$,

$$I=Sy_1=\frac{bh^2}{6}\cdot\frac{h}{2}=\frac{bh^3}{12}.$$

In order to illustrate the degree of accuracy of the approximate method, as explained above, the moment of inertia of a rectangle 2 ins.

in breadth by 12 ins. deep will be computed in accordance therewith, and also by equation (1).

Assuming that the section is divided into twelve strips one inch thick (six above and six below the neutral axis), the area of each strip $= 1 \times 2 = 2$ sq.ins. The distance from the neutral axis to the centre of gravity of the first strip $= 0.5$ in., that to the centre of gravity of the second strip $= 1.5$ ins., to the third strip $= 2.5$ ins., to the fourth strip $= 3.5$ ins., to the fifth strip $= 4.5$ ins. and to the sixth strip $= 5.5$ ins. Then

$$I \text{ (approx.)} = (0.5^2 + 1.5^2 + 2.5^2 + 4.5^2 + 5.5^2) \times 2 \times 2 \text{ sq.ins.} = 286.$$

If, now, the section be divided into twenty-four strips $\frac{1}{2}$ in. thick, the area of each strip $= \frac{1}{2} \times 2 = 1$ sq.in. Then, muitiplying each area by the square of the distance of its centre of gravity from the neutral axis, as before,

$$I \text{ (approx.)} = (0.25^2 + 0.75^2 + 1.25^2 + 1.75^2 + 2.25^2 + 2.75^2 + 3.25^2 + 3.75^2 + 4.25^2 + 4.75^2 + 5.25^2 + 5.75^2) \times 2 \times 1 \text{ sq.in.} = 287.5.$$

By the exact method, equation (1)

$$I = \frac{bh^3}{12} = \frac{2 \times 12^3}{12} = 288.$$

Thus the difference between the first approximation and the exact result is less than three-quarters of one per cent, whereas the second approximation is almost exact.

The approximate method of computing moments of inertia is very suitable in dealing with an irregular section, such as that of a track rail. It is first necessary, however, to determine the position of its centre of gravity, which may be done by cutting the section out of cardboard and balancing over a knife-edge.

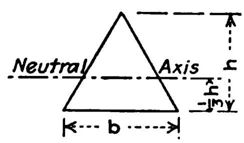


FIG. 26.

For a triangle, Fig. 26, the moment of inertia about an axis through its centre of gravity and parallel to its base, is

$$I = \frac{bh^3}{36}. \qquad (2)$$

For a circle, Fig. 27, the moment of inertia about an axis through its centre is

$$I=\frac{\pi d^4}{64}. \qquad (3)$$

The moment of inertia of a compound section about an axis through its centre of gravity is equal to the sum of the moments of inertia of the component parts about axes through their own centres of gravity, plus the areas of the component parts, multiplied by the squares of the distances of their individual centres of gravity from the neutral axis of the whole figure. Then the moment of inertia for any section composed of rectangles, triangles, or circles may be computed by the aid of the formulæ for the moments of inertia of these three shapes, as given above.

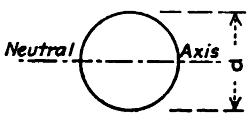


FIG. 27.

ART. 9. CALCULATIONS OF MOMENTS OF INERTIA

EXAMPLE 1. Fig. 28 represents a $6\times3\frac{1}{2}\times\frac{1}{2}$ in. angle, divided into two rectangles: one 6×0.5 in. $=3.0$ sq.ins.; and the other 3×0.5 in. $=1.5$ sq.ins. It is first necessary to find the positions of the axes *ab* and *cd* through the centre of gravity *cg*. For the axis *ab* moments of the areas are taken about the back of the shorter leg; the distance to the centre of gravity of the 3-inch leg being 0.25 in., and that to the centre of gravity of the 6-inch leg being 3 ins.; thus:

Areas.	Levers.	Moments.
1.5 sq.ins. ×	0.25 ins. =	0.375
3.0 sq.ins. ×	3.00 ins. =	9.000
4.5 sq.ins.		9.375

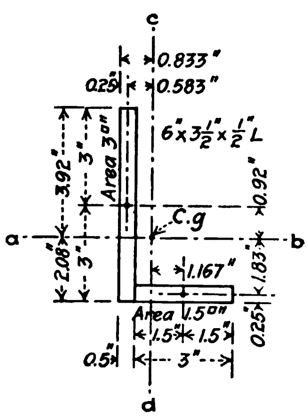


FIG. 28.

Then the total moment, divided by the total area, is the distance from the point of moments to the centre of gravity; or, $9.375\div4.5=2.08$ ins.

For the position of the axis *cd*, moments are taken about the back of the longer leg and divided by the total area, as before.

Areas.	Levers.	Moments.
3.0 sq.ins. ×	0.25 ins. =	0.75
1.5 sq.ins. ×	2.00 ins. =	3.00
4.5 sq.ins.		3.75

Then $3.75 \div 4.5 = 0.833$ ins., which is the distance from the back of the longer leg to the axis *cd*.

Moment of Inertia about Axis ab

For rectangle (6×0.5 in.), $I = \frac{bh^3}{12} = \frac{0.5 \times 6^3}{12} = 9.00$

Area of rectangle into sq. of dist. of its c.g. from axis $= 3.0 \times 0.92^2 = 2.54$

For rectangle (3×0.5 in.), $I = \frac{bh^3}{12} = \frac{3.0 \times 0.5^3}{12} = 0.03$

Area of rectangle into sq. of dist. of its c.g. from axis $= 1.5 \times 1.83^2 = 5.02$

16.59

Moment of Inertia about Axis cd

For rectangle (6×0.5 in.), $I = \frac{bh^3}{12} = \frac{6.0 \times 0.5^3}{12} = 0.06$

Area of rectangle into sq. of dist. of its c.g. from axis $= 3.0 \times 0.583^2 = 1.02$

For rectangle (3×0.5 in.), $I = \frac{bh^3}{12} = \frac{0.5 \times 3^3}{12} = 1.12$

Area of rectangle into sq. of dist. of its c.g. from axis $= 1.5 \times 1.167^2 = 2.04$

4.24

EXAMPLE 2. Fig. 29 represents a 24-inch I-beam at 80 lbs. per foot. As this section is symmetrical, the axes *ab* and *cd* are centre lines, as shown. The figure is divided into rectangles and triangles, from which the area may be computed, as follows:

Area of web $= 22.80 \times 0.5 = 11.40$

Area of flanges $= 7.0 \times 0.6 \times 2 = 8.40$

Area of triangles $= \frac{3.25 \times 0.542}{2} \times 4 = 3.52$

Square inches, 23.32

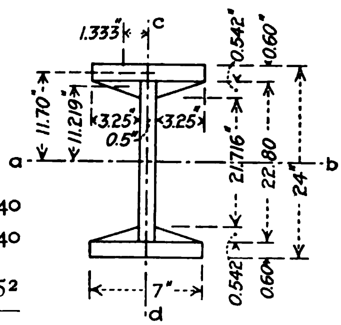


FIG. 29.

Moment of Inertia about Axis ab.

For web,	$I=\frac{bh^3}{12}$	$=\frac{0.5\times 22.80^3}{12}$	$= 493.85$
For flanges,	$I=\frac{bh^3}{12}\times 2$	$=\frac{7.0\times 0.6^3}{12}\times 2$	$= 0.25$
Area of flanges into sq. of dist. of their c.g. from axis		$=8.40\times 11.70^2$	$=1149.88$
For triangles,	$I=\frac{bh^3}{36}\times 4$	$=\frac{3.25\times 0.542^3}{36}\times 4=$	0.02
Area of triangles into sq. of dist. of their c.g. from axis		$=3.52\times 11.219^2$	$= 443.05$
			2087.05

Moment of Inertia about Axis cd

For web,	$I=\frac{bh^3}{12}$	$=\frac{22.8\times 0.5^3}{12}$	$= 0.24$
For flanges,	$I=\frac{bh^3}{12}\times 2$	$=\frac{0.6\times 7.0^3}{12}\times 2$	$= 34.30$
For triangles,	$I=\frac{bh^3}{36}\times 4$	$=\frac{0.542\times 3.25^3}{36}\times 4=$	2.07
Area of triangles into sq. of dist. of their c.g. from axis		$=3.52\times 1.333^2$	$= 6.26$
			42.87

It is seldom necessary for the designer to compute the various properties of angles, beams, channels, etc., as they are given in the handbooks published by the rolling mills companies. In the subsequent examples these books will generally be referred to for the necessary data.

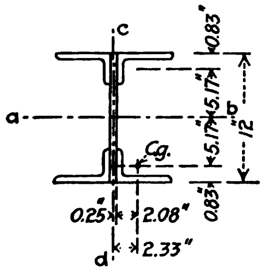


FIG. 30.

EXAMPLE 3. Fig. 30 represents a section composed of

1 web-plate $12\times\frac{1}{2}$ ins. $=$ 6.00
4 angles $6\times 3\frac{1}{2}\times\frac{1}{2}$ ins. $=$ 18.00

24.00 sq.ins.

The section is symmetrical, and the two axes *ab* and *cd* correspond with the centre lines. For the $6\times 3\frac{1}{2}\times\frac{1}{2}$ in. angles, the centre of gravity and moments of inertia have already been computed in Example 1.

Moment of Inertia about Axis ab

For web-plate ($12\times\frac{1}{2}$ in.), $I=\dfrac{bh^3}{12}=\dfrac{0.5\times 12^3}{12}$ $=72.00$

For four angles ($6\times3\frac{1}{2}\times\frac{1}{2}$ in.), I (Example 1) $=4.24\times4$ $=16.96$

Area of angles into sq. of dist. of their c.g. from axis $=18.0\times5.17^2$ $=481.12$

570.08

Moment of Inertia about Axis cd

For web-plate ($12\times\frac{1}{2}$ in.), $I=\dfrac{bh^3}{12}=\dfrac{12.0\times 0.5^3}{12}$ $=0.12$

For four angles ($6\times3\frac{1}{2}\times\frac{1}{2}$ in), I (Example 1) $=16.59\times4$ $=66.36$

Area of angles into sq. of dist. of their c.g. from axis $=18.0\times2.33^2$ $=97.72$

164.20

EXAMPLE 4. Fig. 31 represents a chord section composed of

Two 15″ channels at 33 lbs. $=19.80$
One $24\times\frac{1}{2}$ in. cover-plate $=12.00$

Sq.ins. 31.80

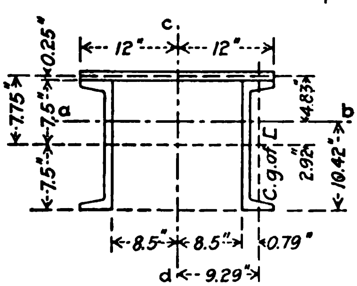


FIG. 31.

To find the position of the axis *ab* through the centre of gravity of the section, the simplest method is to take moments of the areas about the centre of gravity of the channels, and divide by the total area. The result will be the distance from the centre of gravity of the channels to the required axis, thus:

	Areas.	Levers.	Moments.
Channels	19.8 sq.ins.	$\times$0.0 ins.	$=$ 0
Plate	12.0 sq.ins.	$\times$7.75 ins.	$=$93
	31.8 sq.ins.		93

Then $93\div31.8=2.92$ ins., which is the required distance.

The axis *cd* corresponds with the centre line of the section.

Moment of Inertia about Axis ab

For $24 \times \frac{1}{2}$ in. plate, $I = \frac{bh^3}{12} = \frac{24 \times 0.5^3}{12} = 0.25$

Area of plate into sq. of dist. of its c.g. from axis $= 12.0 \times 4.83^2 = 279.95$

For two 15-in. channels (from Carnegie), $I = 312.6 \times 2 = 625.20$

Area of channels into sq. of dist. of their c.g. from axis $= 19.8 \times 2.92^2 = 168.82$

1074.22

Moment of Inertia about Axis cd

For $24 \times \frac{1}{2}$ in. plate, $I = \frac{bh^3}{12} = \frac{0.5 \times 24^3}{12} = 576.00$

For two 15-in. channels (from Carnegie), $I = 8.23 \times 2 = 16.46$

Area of channels into sq. of dist. of their c.g. from axis $= 19.8 \times 9.29^2 = 1708.82$

2301.28

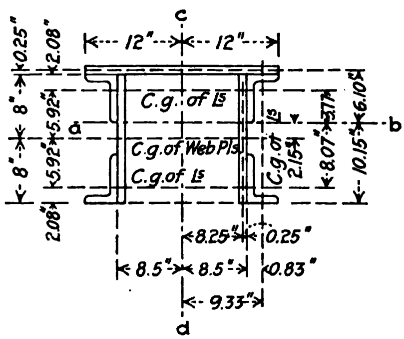


FIG. 32.

EXAMPLE 5. Fig. 32 represents a chord section composed of

1 cover-plate	$24 \times \frac{1}{2}$ in.	$= 12.00$
2 web-plates	$16 \times \frac{1}{2}$ in.	$= 16.00$
4 angles	$6 \times 3\frac{1}{2} \times \frac{1}{2}$ in.	$= 18.00$
	Sq.ins.	46.00

To find the position of the axis *ab* through the centre of gravity of the section, moments of the areas are taken about the c.g. of the web-plates, thus:

	Areas.	Levers.	Moments.
Web-plates and angles	34.0 sq.ins.	$\times$ 0.0 ins.	= 0
Cover plate	12.0 sq.ins.	$\times$ 8.25 ins.	= 99
	46.0 sq.ins.		99

Then $99 \div 46 = 2.15$ ins., which is the distance from the centre of gravity of the web-plates to the axis *ab*. The axis *cd* corresponds with the centre line of the section.

Moment of Inertia about Axis ab

For $24\times\frac{1}{2}$ in. cover-plate, $I=\dfrac{bh^3}{12}$	$=\dfrac{24\times 0.5^3}{12}$	$=$ 0.25
Area of cover-plate into sq. of dist. of its c.g. from axis	$=12.0\times 6.1^2$	$=$ 446.52
For two $16\times\frac{1}{2}$ in. web-plates, $I=\dfrac{bh^3}{12}\times 2$	$=\dfrac{0.5\times 16^3}{12}\times 2$	$=$ 341.33
Area of web-plates into sq. of dist. of their c.g. from axis	$=16.0\times 2.15^2$	$=$ 73.96
For four $6\times 3\frac{1}{2}\times\frac{1}{2}$ in. angles, I (Ex. 1)	$=16.59\times 4$	$=$ 66.36
Area of upper angles into sq. of dist. of their c.g. from axis	$=9.0\times 3.77^2$	$=$ 127.92
Area of lower angles into sq. of dist. of their c.g. from axis	$=9.0\times 8.07^2$	$=$ 586.12
		1642.46

Moment of Inertia about Axis cd

For $24\times\frac{1}{2}$-in. cover-plate, $I=\dfrac{bh^3}{12}$	$=\dfrac{0.5\times 24^3}{12}$	$=$ 576.00
For two $16\times\frac{1}{2}$ in. web-plates, $I=\dfrac{bh^3}{12}\times 2$	$=\dfrac{16.0\times 0.5^3}{12}\times 2$	$=$ 0.33
Area of web-plates into sq. of dist. of their c.g. from axis	$=16.0\times 8.25^2$	$=$ 1089.00
For four $6\times 3\frac{1}{2}\times\frac{1}{2}$ in. angles, I (Ex. 1)	$=4.24\times 4$	$=$ 16.96
Area of angles into sq. of dist. of their c.g. from axis	$=18.0\times 9.33^2$	$=$ 1566.88
		3249.17

Art. 10. Radius of Gyration

The radius of gyration of a section, which is usually represented by r, is the distance from the neutral axis through the centre of gravity to a point where, if the total area of the section could be concentrated and multiplied by the square of this distance, the result would be the moment of inertia of the actual cross-section; thus, representing the area by A,

$$I = Ar^2, \quad \text{and therefore} \quad r = \sqrt{\frac{I}{A}}. \quad . \quad . \quad . \quad . \quad (1)$$

The radius of gyration is a quantity used principally in connection with formulæ for the strength of columns and struts, and will be referred to frequently in subsequent chapters.

For a $6 \times 3\frac{1}{2} \times \frac{1}{2}$ in. angle, Fig. 28, Art. 9, $A = 4.5$ sq.ins., I(axis ab) $= 16.59$, I(axis cd) $= 4.24$; then $r(ab) = \sqrt{\frac{16.59}{4.5}} = 1.92$ ins., $r(cd) = \sqrt{\frac{4.24}{4.5}} = 0.97$ in.

For a 24-in. I-beam at 80 lbs., Fig. 29, $A = 23.33$ sq.ins., $I(ab) = 2087.05$, $I(cd) = 42.87$; then $r(ab) = \sqrt{\frac{2087.05}{23.33}} = 9.46$ ins. $r(cd) = \sqrt{\frac{42.87}{23.33}} = 1.36$ ins.

For the compound section, Fig. 30, $A = 24.0$ sq.ins., $I(ab) = 570.08$, $I(cd) = 164.20$; then $r(ab) = \sqrt{\frac{570.08}{24.0}} = 4.88$ ins., $r(cd) = \sqrt{\frac{164.20}{24.0}} = 2.62$ ins.

For the compound section, Fig. 31, $A = 31.8$ sq.ins., $I(ab) = 1074.22$, $I(cd) = 2301.28$; then $r(ab) = \sqrt{\frac{1074.22}{31.8}} = 5.81$ ins., $r(cd) = \sqrt{\frac{2301.28}{31.8}} = 8.51$ ins.

For the compound section, Fig. 32, $A = 46.0$ sq.ins., I $(ab) = 1642.46$, $I(cd) = 3249.17$; then $r(ab) = \sqrt{\frac{1642.46}{46.0}} = 5.98$ ins., $r(cd) = \sqrt{\frac{3249.17}{46.0}} = 8.40$ ins.

Art. 11. Formulæ Relating to Beams

The following notation and relations between bending moments and the various properties of beams, which have already been referred to in Arts. 7, 8, 9 and 10, are here set forth more concisely. The student should familiarize himself with the formulæ, as they will be referred to frequently in the following pages.

M = bending moment in inch-pounds;
I = moment of inertia of section about axis through centre of gravity;
f_1 = stress in lbs. per sq.in. on outer fibres;
y_1 = distance in inches from centre of gravity of section to outer fibres, where unit-stress $= f_1$;
f = stress in lbs. per sq.in. on any fibres;
y = distance in inches from centre of gravity of section to fibres where unit stress $= f$;
S = section modulus;
A = area of section, in sq.ins.;
r = radius of gyration, in inches.

Then

$$M = \frac{I}{y_1} f_1 = S f_1; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$S = \frac{M}{f_1} = \frac{I}{y_1}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$f_1 = \frac{M}{S} = \frac{M y_1}{I}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$f = \frac{M y}{I}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$I = A r^2; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$r^2 = \frac{I}{A}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$r = \sqrt{\frac{I}{A}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Art. 12. Distribution of Shearing Stresses in Beams

It has been shown in Art. 7. that, when a beam is loaded transversely, the fibres on one side of the neutral axis are compressed, while those on the other side are extended; and that the intensity of the fibre-stress at any point is equal to the bending moment (in inch-pounds), multiplied by the distance in inches of the point from the neutral axis, and divided by the moment of inertia of the section; or

$$f=\frac{My}{I}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now, considering a simple beam, supported at both ends and subjected to the action of a vertical load, the bending moment M increases from the ends towards the centre of the span, and with it the intensity f of the horizontal stresses. Thus f varies both vertically and horizontally, as illustrated in Fig. 33, where the two sections o and x are supposed to be very close together.

The fibre-stresses at section o are represented by fo, and those at section x are represneted by fx, while the longitudinal variations of the fibre-stresses between these two sections are indicated, at section x, by the cross-shaded areas. This increase of horizontal stress from one section to another induces a force at every longitudinal layer tending to slide it past the next section above it; and this sliding or shearing force, which increases at every layer, is transmitted towards the neutral axis, where it attains its maximum intensity. This stress is called the longitudinal shear and may be deduced as follows:

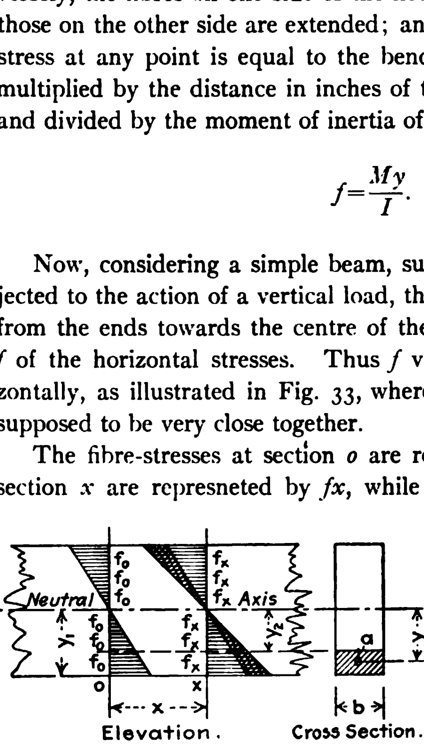


Fig. 33.

At the longitudinal layer distant y_2 from the neutral axis, the total horizontal stresses at sections o and x between this layer and the extreme fibres are equal respectively to

$$\sum_{y_2}^{y_1} f_0 a \quad \text{and} \quad \sum_{y_2}^{y_1} f_x a,$$

in which a equals an infinitely small area of cross-section, distant y from the neutral axis. Thus the longitudinal shear between sections o and x at the layer y_2 is equal to

$$\sum_{y_2}^{y_1} f_x a - \sum_{y_2}^{y_1} f_0 a.$$

Substituting in this expression the values of fo and fx, as given by equation (1), the following is obtained:

$$\sum_{y_2}^{y_1} f_x a - \sum_{y_2}^{y_1} f_0 a = \left(\frac{M_x - M_0}{I}\right) \sum_{y_2}^{y_1} ay,$$

M_0 and M_x being the moments at o and x respectively. This longitudinal shear is acting on an area equal to the breadth b of the cross-section at the layer y_2, multiplied by the distance x between the sections o and x, $= bx$. Therefore, if the intensity of the shear be represented by t,

$$t = \left(\frac{M_x - M_0}{I}\right) \sum_{y_2}^{y_1} ay \div bx = \left(\frac{M_x - M_0}{Ibx}\right) \sum_{y_2}^{y_1} ay. \quad . \quad . \quad . \quad (2)$$

In addition to the longitudinal shear, as above, there co-exists a vertical shear which is equal to the algebraic sum of all the external vertical forces acting on either side of the vertical plane considered, as demonstrated in Art. 6; and, in order that any particle of the beam be in equilibrium, the intensities of the vertical and horizontal shears acting on it must be equal. This may be proven as follows:

Fig. 34 represents an infinitely small portion of the side of the beam, at a point distant Y_2 from the neutral axis. The sides of the element are both represented by d, and the breadth of the beam at this point by b. Now there are two shearing stresses acting upon it, one vertical and the other horizontal; and these two shears form two pairs of couples, acting as indicated by the four vertical and horizontal arrows. If the intensity of the horizontal shear at this point be represented by t_x, and that of the vertical shear by t_y, then the total horizontal and vertical shears acting on the element are equal respectively to

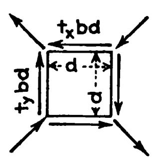


FIG. 34.

$$t_x bd \quad \text{and} \quad t_y bd;$$

and, in order that the body be in equilibrium, the moments of these two shears must be equal, thus:

$$(t_x bd)d = (t_y bd)d,$$

$$\therefore \qquad t_x = t_y.$$

Since the intensities of the horizontal and vertical shears are equal, they will both be represented by the common symbol t.

As stated in Art. 6, the bending moment at any point of a loaded beam is equal to the algebraic sum of the moments, about this point, of the external forces on either side of it. Then, since sections o and x (Fig. 33) are assumed to be very close together, the direct load between them may be neglected; and the moment at section x will be equal to that at section o, plus the shear at section o multiplied by the distance between the two sections, thus:

$$M_x = M_0 + Qx;$$

in which Q = the vertical shear at o,

x = the distance between sections o and x.

$$\therefore \qquad Q = \frac{M_x - M_0}{x}. \qquad (3)$$

The substitution of this value of $\frac{M_x - M_0}{x}$ in equation (2) gives

$$t = \frac{Q}{Ib} \sum_{y_2}^{y_1} ay. \qquad (4)$$

Therefore, the intensity of the shearing stress at any point of a solid beam of any section is equal to the total vertical shear at the point, multiplied by the moment, about the neutral axis, of that portion of the cross-section which lies outside of the longitudinal plane considered, and divided by the product of the moment of inertia of the entire cross-section into the breadth of the section at this point.

For a beam of rectangular cross-section, the intensity of the shear at the top and bottom is zero, and maximum at the neutral axis, where it is equal to one and one-half times that of the mean intensity for the whole section; the intensities of shear at intermediate points being

represented by the ordinates to a parabola, having its vertex in the neutral axis, as shown in Fig. 35.

Taking as an example a rectangular beam 12 inches deep and 2 inches wide, the moment of inertia of which is 288, and assuming that the total vertical shear on the section is 12,000 lbs., the intensities of shear at various points will be computed from equation (4), as follows:

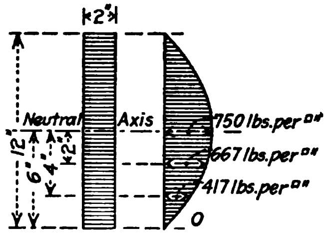


FIG. 35.

The area of that portion of the section outside of a horizontal plane taken 4 inches from the neutral axis $=2\times 2=4$ sq.ins., and the distance of its c.g. from the axis $=5$ ins.; thus the moment of this area about the neutral axis $=4\times 5=20$. Then, at a distance of 4 inches from the neutral axis,

$$t=\frac{12{,}000}{288\times 2}\times 20=417 \text{ lbs. per sq.in.}$$

The area outside of a horizontal plane 2 inches from the neutral axis $=4\times 2=8$ sq.ins., the distance of its c.g. from axis $=4$ ins., and its moment about the neutral axis $=8\times 4=32$. Then, at a distance of 2 inches from the neutral axis,

$$t=\frac{12{,}000}{288\times 2}\times 32=667 \text{ lbs. per sq.in.}$$

The area outside of the neutral axis $=6\times 2=12$ sq.ins., the distance of its c.g. from axis $=3$ ins., and its moment about the neutral axis $=12\times 3=36$. Then, at the neutral axis,

$$t=\frac{12{,}000}{288\times 2}\times 36=750 \text{ lbs. per sq.in.}$$

The average shear on the whole section $=\frac{12{,}000}{24}=500$ lbs. per sq.in.; and thus the intensity of the shear at the neutral axis $=500\times 1\frac{1}{2}=750$ lbs. per sq.in.; or one and one-half times the average intensity, as before stated.

As a further illustration of the distribution of shear in beams, the intensities of shear will now be computed for a 24-inch I-beam at 80 lbs. per foot, as shown in Fig. 36.

The assumed vertical shear on the section = 72,000 lbs.; the moment of inertia of the section (Art. 9, Example 2) = 2,087; the area of that portion of the section outside of a horizontal plane distant 11.40 ins. from the neutral axis = 4.20 sq.ins., and its moment about the neutral axis = 49.14; the area outside of a horizontal plane 10.86 ins. from the neutral axis = 6.23 sq.ins., and its moment about the neutral axis = 71.73; the area outside of the neutral axis = 11.66 sq.ins.; and its moment about the neutral axis = 101.21. Then,

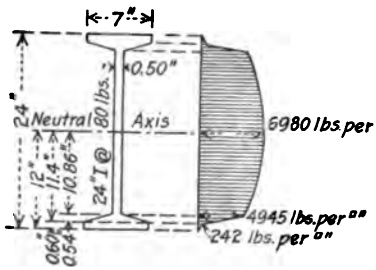


FIG. 36.

At a distance of 11.4 ins. from neutral axis,

$$t = \frac{72,000}{2,087 \times 7} \times 49.14 = 242 \text{ lbs. per sq.in.}$$

At a distance of 10.86 ins. from neutral axis,

$$t = \frac{72,000}{2,087 \times 0.5} \times 71.73 = 4,945 \text{ lbs. per sq.in.}$$

At the neutral axis,

$$t = \frac{72,000}{2,087 \times 0.5} \times 101.21 = 6,980 \text{ lbs. per sq.in.}$$

Now the area of the web (which is 21.72 ins. deep and 0.50 in. thick) = 10.86 sq.ins.; and, assuming that it takes all the shear, the average intensity of this stress = 72,000 ÷ 10.86 = 6,630 lbs. per sq.in., which is only 5 per cent less than the maximum. In the case of a plate-girder having a comparatively thin web, it will be found that the difference between the maximum and the average shears is still less. It is therefore customary to assume that the shear is distributed evenly over the web.

As demonstrated above, in connection with Fig. 34, there exist at every point of the web two shearing stresses of equal intensity, the one

vertical and the other horizontal. Now the effect of these shearing stresses is to produce two direct stresses, of the same intensity as the shearing stresses, and at angles of 45° with the neutral axis, as indicated by the diagonal arrows. One of these direct stresses is tension and the other compression.

ART. 13. SIZES OF BEAMS REQUIRED FOR VARIOUS CASES

In the following examples the maximum permissible fibre-stress will be taken at 16,000 lbs. per sq.in., which is about one-half the elastic limit (in tension) for mild steel; and the average permissible shear on the web, at 10,000 lbs. per sq.in. The section moduli S will be taken from Carnegie's handbook. Unless supported laterally, no beam should be used when the span exceeds twenty times the width of the flange; as the compression flange of a beam is liable to fail by crippling sidewise. Floorbeams and stringers in buildings, however, are usually supported at much closer intervals, either by other beams framed thereto, or by the floor resting directly on them. Since the section modulus of a beam is computed in inches, the bending moment must always be reduced to inch-pounds.

EXAMPLE 1. A beam of 20 ft. span is required to support a uniform load of 1,600 lbs. per lin.ft., which is supposed to include the weight of the beam. This case is similar to that illustrated in Fig. 18, Art. 6.

The maximum end shear $=\frac{wl}{2}=\frac{1,600\times 20}{2}=16,000$ lbs., and $\frac{16,000}{10,000}$ $=1.6$ sq.ins. required in the web.

The maximum moment at the centre $=\frac{wl^2}{8}=\frac{1,600\times 20^2}{8}=80,000$ ft.-lbs. $=960,000$ in.-lbs., and $S=\frac{M}{f_1}$ (Art. 11) $=\frac{960,000}{16,000}=60$.

Turning to Carnegie's table of "Properties of Standard I-Beams," it will be found that a 15-in. beam weighing 45 lbs. per ft. conforms most closely with these requirements. Its section modulus $=60.8$; and the area of its web $=13\times 0.46=5.98$ sq.ins. The area of the web greatly exceeds that required for the maximum shear; and this will usually be the case, except for short spans carrying very heavy loads.

EXAMPLE 2. A beam of 16 ft. span is required to support, in addition to its own weight, a concentrated load at the centre of 40,000 lbs.

Since the size of the beam is unknown, its weight can only be assumed. It will be taken at 70 lbs. per lin.ft.

The moment at the centre $=\frac{wl^2}{8}+\frac{Pl}{4}=\frac{70\times16^2}{8}+\frac{40,000\times16}{4}=$ $2,240+160,000=162,240$ ft.-lbs. $=1,946,880$ in.-lbs. Then $S=\frac{1,946,880}{16,000}$ $=121.7$. A 20-inch I-beam weighing 70 lbs. per foot is the proper one to use. Its section modulus $=122.0$.

Example 3. A cantilever beam projecting 10 ft. from a wall is required to carry, in addition to its own weight, a uniform load of 800 lbs. per lin.ft.

Assuming the beam to weigh 50 lbs. per lin.ft., the maximum moment at the point of support $=\frac{wl^2}{2}=\frac{850\times10^2}{2}=42,500$ ft.-lbs. $=$ $510,000$ in.-lbs.; and $\frac{510,000}{16,000}=31.8$, which is the section modulus required. A 12-inch I-beam weighing 31.5 lbs. per foot, and with $S=36$, will suit this case.

Example 4. A 15-in. I-beam at 42 lbs. per foot projects 8 ft. from a wall. It is required to know what load can safely be placed at its outer end. Now $S=58.9$, and the maximum moment $=58.9\times16,000$ $=942,400$ in.-lbs. $=78,533$ ft.-lbs. The moment due to the weight of the beam $=\frac{wl^2}{2}=\frac{42\times8^2}{2}=1344$ ft.-lbs. Then $Pl=78,533-1,344=77,189$ ft.-lbs. Whence $P=\frac{77,189}{8}=9,648$ lbs.

CHAPTER III

ART. 14. DEFLECTION OF BEAMS

IT has already been demonstrated in Art. 7 that, when a beam is loaded transversely, it bends about the neutral axis which lies in the centre of gravity of its section; and that the fibres on one side of this neutral axis are compressed, while those on the other side are extended; the fibres furthest from the neutral axis being distorted most, and the intermediate fibres in direct proportion to their distance from the axis.

Fig. 37 represents a portion of a bent beam, the curve being greatly exaggerated. The two planes cutting the neutral axis at A and B, which are assumed to be very close together, were parallel before bending occurred, but will now meet in the point C.

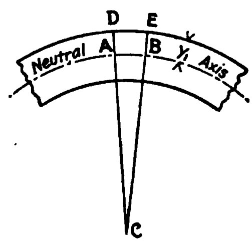


FIG. 37.

Then CA or $CB=R=$radius of curvature of the arc AB;

$AB=$original length of beam between the two planes considered;

$DE=$strained length at the outer fibres;

$\frac{DE-AB}{AB}=$unit-strain at the outer fibres.

Now, from similar triangles, $\frac{DC}{AC}=\frac{EC}{BC}=\frac{DE}{AB}=\frac{R+y_1}{R}$; from which is obtained $\frac{y_1}{R}=\frac{DE-AB}{AB}=$unit-strain at outer fibres, as before. But, as explained in Art. 1, the modulus of elasticity $E=\frac{\text{unit-stress}}{\text{unit-strain}}$; therefore, since the unit-stress at the outer fibres$=f_1$, the unit-strain

at the outer fibers $=\frac{f_1}{E}$; and, as has been shown in Art. 7, $f_1=\frac{My_1}{I}$; consequently, $\frac{y_1}{R}=\frac{f_1}{E}=\frac{My_1}{EI}$; from which

$$\frac{1}{R}=\frac{M}{EI}. \qquad (1)$$

In Fig. 38 AB represents a deflected beam, built firmly into the wall at B. The shaded area above the beam is the moment diagram; and δx is an element thereof, distant x from A. From the ends of this element, normals are drawn to the beam, intersecting in the point C, which is the centre of curvature for the element; and the radius of curvature $=R$. Also, from the ends of the element, tangents are drawn to the beam, intersecting the vertical line through A, the intercept being represented by Δ. Then, if $\delta\phi$ be the angle at C, it is also the angle between the two tangents; and

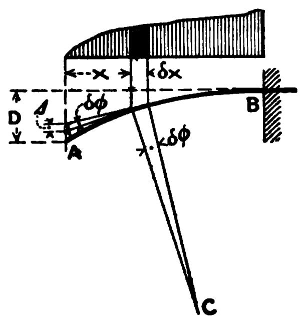


FIG. 38.

$$\Delta=x\cdot\delta\phi.$$

Now $\delta x=R\cdot\delta\phi$; from which $\delta\phi=\frac{\delta x}{R}$; and, by equation (1), $\frac{1}{R}=\frac{M}{EI}$ (in which M = moment at x); therefore,

$$\Delta=\frac{x\cdot\delta x}{R}=x\cdot\delta x\cdot\frac{M}{EI}.$$

But $M\cdot\delta x$ = area of element of moment diagram; consequently, the total deflection

$$D=\Sigma x\cdot\delta x\cdot\frac{M}{EI}=\frac{AX}{EI}, \qquad (2)$$

in which A = total area of moment diagram,

X = distance of its centre of gravity from free end of beam.

The above theory will now be applied in determining deflection formulæ for various cases, in all of which

l = length of beam in inches;

P = a concentrated load, in lbs.;

w = intensity of load in lbs. per lineal inch.

It should be noted that, since the moment of inertia is computed in inches, all other linear dimensions must also be in inches.

CASE I. Beam fixed at one end and loaded at the other end. Fig. 39.

As demonstrated in Art. 6, the curve of the moment diagram is a straight line with maximum ordinate at the fixed end of beam equal to Pl. The form of the moment area being a triangle, its area is equal to $\frac{Pl^2}{2}$, and the distance of its centre of gravity from the free end of beam is equal to $\frac{2}{3}l$. Then

$$D=\frac{AX}{EI}=\frac{Pl^2}{2}\times\frac{2}{3}l\times\frac{1}{EI}=\frac{Pl^3}{3EI}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

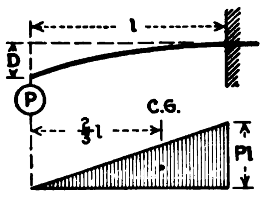


FIG. 39.

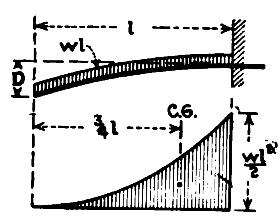


FIG. 40.

CASE II. Beam fixed at one end and uniformly loaded. Fig. 40.

Here the curve of the moment diagram is a parabola with vertex at the free end of beam, and maximum ordinate at the fixed end equal to $\frac{wl^2}{2}$. Since the area of a parabolic segment is equal to two-thirds that of the enclosing rectangle, the moment area is equal to $\frac{wl^2}{2}\times\frac{l}{3}$ $=\frac{wl^3}{6}$. The distance of its centre of gravity from the free end of beam is equal to $\frac{3}{4}l$. Then

$$D=\frac{AX}{EI}=\frac{wl^3}{6}\times\frac{3}{4}l\times\frac{1}{EI}=\frac{wl^4}{8EI}. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

CASE III. Beam supported at both ends, with a single load at the middle point. Fig. 41.

On account of symmetry of loading, the beam will be horizontal at the middle, and the deflection at this point will be equal to the elevation of either support above the horizontal tangent, and the curve of the beam at either side of the middle point will be similar, but in opposite direction, to that of Case I. Now the maximum ordinate of the moment-diagram equals $\frac{Pl}{4}$. The form of the moment area for one-half of the span is a triangle, its base being equal to $\frac{l}{2}$, and its altitude to $\frac{Pl}{4}$. Then $A=\frac{Pl}{4}\times\frac{l}{4}=\frac{Pl^2}{16}$; $X=\frac{1}{2}\times\frac{2}{3}l=\frac{1}{3}l$; and

$$D=\frac{AX}{EI}=\frac{Pl^2}{16}\times\frac{1}{3}l\times\frac{1}{EI}=\frac{Pl^3}{48EI}. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

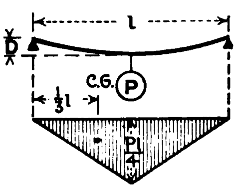


FIG. 41.

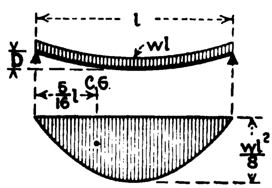


FIG. 42.

CASE IV. Beam supported at both ends and uniformly loaded. Fig. 42.

As in Case III, the deflection at the middle of the beam will be equal to the elevation of either support above the horizontal tangent to the curve at this point. The form of the moment-diagram is a parabola, the maximum ordinate between it and the closing line being equal to $\frac{wl^2}{8}$. The moment area A at either side of the centre line is equal to two-thirds of the enclosing rectangle for the semi-segment of the parabola, $=\frac{wl^2}{8}\times\frac{1}{2}l\times\frac{2}{3}=\frac{wl^3}{24}$; and the distance X of its centre of gravity from the end of beam $=\frac{5}{8}\times\frac{1}{2}l=\frac{5}{16}l$. Then

$$D=\frac{AX}{EI}=\frac{wl^3}{24}\times\frac{5}{16}l\times\frac{1}{EI}=\frac{5wl^4}{384EI}. \quad . \quad . \quad . \quad . \quad . \quad (6)$$

CASE V. Beam supported at both ends, carrying a single load at any point. Fig. 43.

For this case it will be more satisfactory to consider a numerical example, taking the length, centre to centre of bearings, equal to 240 ins., and a load of 16,000 lbs., placed 60 ins. from the left-hand end. The maximum moment which is at the point of loading = $\frac{16{,}000 \times 180 \times 60}{240} = 720{,}000$ in.-lbs.; and $\frac{720{,}000}{16{,}000} = 45.0$, which is the section modulus required. A 12-inch at 40 lbs. I-beam will be used, for which $S = 44.8$ and $I = 269$.

Before computing the deflection, it is necessary to determine the point where a horizontal line is tangent to the curve of the beam. Since the curvature of a beam at any point is proportional to the bending moment thereat, and since the total curvature on one side of the point of tangent must be equal to that on the other, therefore, the moment area on one side of this point of tangent, multiplied by the distance of its centre of gravity from the adjacent support, must be equal to the moment area on the other side, into the distance of its centre of gravity from the other support. Now the area of the whole moment-diagram $= \frac{720{,}000 \times 240}{2} = 86{,}400{,}000$, and the distance of its centre of gravity from the left-hand support $= \frac{60 + 240}{3} = 100$ ins.; the area on the length $x = \frac{720{,}000x^2}{180 \times 2} = 2{,}000x^2$, which will be denoted by A, and the distance of its centre of gravity from the right-hand support $= \frac{2}{3}x$, while that from the left-hand support $= 240 - \frac{2}{3}x$. Then

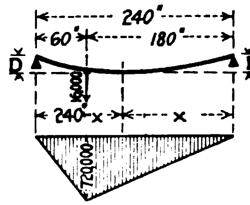


FIG. 43.

$$2000x^2 \cdot \tfrac{2}{3}x = (86{,}400{,}000 \times 100) - 2000x^2(240 - \tfrac{2}{3}x).$$

From which $\qquad x^2 = 18{,}000 \quad$ and $\quad x = 134.17$ ins.

The area $A = 2000x^2 = 2000 \times 134.17^2 = 36{,}003{,}178$; and the distance X of its centre of gravity from the right-hand support $= \frac{2}{3}x = 89.45$ ins. Finally,

$$D = \frac{AX}{EI} = \frac{36{,}003{,}178 \times 89.45}{29{,}000{,}000 \times 269} = 0.412 \text{ in.}$$

CASE VI. Beam fixed at both ends, supporting a single load at the middle point. Fig. 44.

Since the beam is fixed at the ends, the tangent to the curve at these points will be horizontal; and, from symmetry of loading, the tangent to the curve at the middle point will also be horizontal. Therefore, between the middle point of the beam and either end, the total curvature in one direction will be equal to that in the other; and, since the angular deflection of a beam is proportional to its moment area, the area between the middle and either point of contraflexure must be equal to the area between one of these points of contraflexure and the near end.

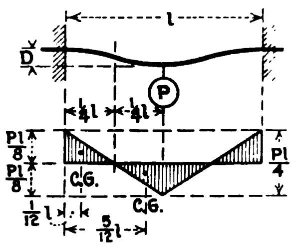


FIG. 44.

Now, for a beam simply supported at the ends and loaded in the middle, the moment-diagram would be a triangle with its base equal to l and its altitude equal to $\frac{Pl}{4}$, the centre moment. In the present case, the positive moment at the centre is reduced by the amount of the negative moment at either end; and, since the positive and negative moment areas must be equal, the moment at the centre and the moment at the ends are each equal to $\frac{Pl}{8}$; thus the points of contraflexure, where the moment is zero, are half-way between the middle point of the beam and either end. Now the positive and negative moment areas A, between the middle point of the beam and either end, are each equal to $\frac{Pl}{8}\times\frac{1}{4}l\times\frac{1}{2}=\frac{Pl^2}{64}$. The distance X of the centre of gravity of the former from the end support $=\frac{1}{4}l+(\frac{2}{3}\times\frac{1}{4}l)=\frac{5}{12}l$; while that of the latter, which will be designated by X_1, $=\frac{1}{3}\times\frac{1}{4}l=\frac{1}{12}l$. Then the deflection at the middle point below either end will be obtained by taking moments of these areas about one end, and dividing by EI, thus:

$$D=\frac{AX-AX_1}{EI}=\left[\left(\frac{Pl^2}{64}\times\frac{5}{12}l\right)-\left(\frac{Pl^2}{64}\times\frac{1}{12}l\right)\right]\frac{1}{EI}=\frac{Pl^3}{192EI}. \quad (7)$$

CASE VII. Beam fixed at both ends, supporting a uniform load. Fig. 45.

For a beam simply supported at the ends and uniformly loaded,

the moment diagram would be a parabola with vertex at the centre of span and depth equal to $\frac{wl^2}{8}$. Now, since the area of the parabolic segment is equal to two-thirds of the enclosing rectangle, it is also equal to a rectangle of length equal to l and depth equal to $\frac{2}{3}\times\frac{wl^2}{8}=\frac{wl^2}{12}$; and, if this rectangle be superimposed on the parabolic segment as shown, the shaded areas at the ends, which represent the negative moments, will be equal to the remaining portion of the parabolic segment below the rectangle $\left(\text{of depth equal to } \frac{wl^2}{8}-\frac{wl^2}{12}=\frac{wl^2}{24}\right)$, which represents the positive moments. Thus the moment at the ends $=-\frac{wl^2}{12}$, and that at the centre of the span $=+\frac{wl^2}{24}$.

If $x=$ the distance from the centre of the span to the point of contraflexure, then $\frac{wl^2}{8}:\left(\frac{l}{2}\right)^2::\frac{wl^2}{24}:x^2$. From which $x^2=\frac{l^2}{12}$ and $x=\frac{l}{\sqrt{12}}$ $=0.289l$.

The deflection at the centre may be obtained by taking moments (about either end) of the positive and negative moment areas between that end and the centre of the span, and dividing by EI, as in Case VI. But, since the original parabolic segment and the superimposed rectangle overlap, the covered area (not shaded) is common to both; and the resultant moment of these areas will be the same as for the shaded areas alone. The area of the semi-parabolic segment, as well as of

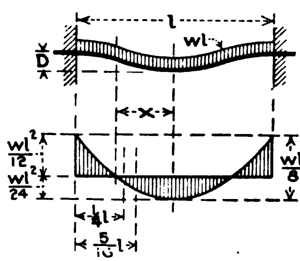


FIG. 45.

the superimposed rectangle, $=\frac{wl^2}{8}\times\frac{1}{2}l\times\frac{2}{3}=\frac{wl^3}{24}$; the distance from end of span to the centre of gravity of the former $=\frac{5}{8}\times\frac{1}{2}l=\frac{5}{16}l$; and to the centre of gravity of the latter $\frac{1}{4}l$. Then

$$D=\frac{wl^3}{24}\left(\frac{5}{16}l-\frac{1}{4}l\right)\frac{1}{EI}=\frac{wl^4}{384EI}. \quad \dots\dots \quad (8)$$

CASE VIII. Beam fixed at one end and supported at the other, carrying a single load at any point. Fig. 46.

If the beam were simply supported at both ends, the moment-diagram would be a triangle of base equal to l, and altitude equal to the bending moment at point of loading $= P\frac{l-a}{l}a = M$. But, since the beam is fixed at the right-hand end, there will be a negative bending moment at this point, represented by the ordinate M_0; and, if a closing line be drawn from the lower extremity of M_0 to the left-hand end of the moment-diagram, the moment at any point will be equal to the ordinate between the closing line and the moment-diagram at that point. The shaded area below this line represents the positive moments, and that above it the negative moments. Now, since the tangent to the curve of the beam at the fixed end is horizontal, it must pass through the supported end; therefore, the summation of the several deflections from the fixed to the supported end must be zero; or the positive moment area, multiplied by the distance of its centre of gravity from the supported end, must be equal to the negative moment area multiplied by the distance of its centre of gravity from the same point. But the two triangles, having the same base l and altitudes of $M = P\frac{l-a}{l}a$ and M_0, overlap one another, and the covered area (not shaded) is common to both; hence the moments of these triangles about the supported end of the beam will also be equal to one another. Now the area of the first triangle $= \left(P\frac{l-a}{l}a\right)\frac{l}{2} = \frac{Pa(l-a)}{2}$, and the distance of its centre of gravity from the supported end $= \frac{a+l}{3}$; the area of the second triangle $= M_0\frac{l}{2}$, and the distance of its centre of gravity from the supported end $= \frac{2}{3}l$. Then,

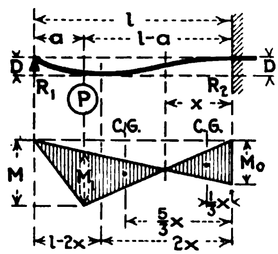


FIG. 46.

$$\frac{Pa(l-a)}{2} \times \frac{a+l}{3} = M_0\frac{l}{2} \times \frac{2}{3}l,$$

from which

$$M_0 = -\frac{Pa(l^2-a^2)}{2l^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

The moment at the point of loading, $M_1 = M - M_0\frac{a}{l}$,

$$= P\frac{l-a}{l}a - \frac{Pa(l^2-a^2)}{2l^2} \times \frac{a}{l}.$$

The point of contraflexure is at the intersection of the closing line with the moment-diagram; and the distance x of this point from the fixed end may be determined by the equation

$$\frac{M_0(l-x)}{l} = \frac{Mx}{l-a},$$

from which

$$x = \frac{M_0 l(l-a)}{Ml + M_0(l-a)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Calling the reaction at the supported end R_1, and that at the fixed end R_2; then $R_1 = P - R_2$, and $R_2 = \frac{M_0}{x}$.

The location of the point of maximum deflection may be determined by the condition that the moment area on one side of this point, multiplied by the distance of its centre of gravity from the adjacent support, must be equal to the moment area on the other side into the distance of its centre of gravity from the other support. When the distance from the fixed end of the beam to the load P is equal to or greater than twice the distance to the point of contraflexure, as in Fig. 46, the positive and negative moment areas on this side of the point of maximum deflection are equal triangles, and the distance to the point required $= 2x$. On the other hand, when the load P is less than $2x$ from the fixed end, as in Fig. 47, the distance x_1 to the point of maximum deflection will be computed by the above rule.

For the condition illustrated in Fig. 46 the maximum deflection will be determined by taking moments, about the fixed end of the beam, of the positive and negative moment areas between the point of maximum deflection and this end, and dividing by EI. Each area is equal to $M_0\frac{x}{2}$, while the distances to the centres of gravity of the positive and negative areas are respectively $\frac{5}{3}x$ and $\frac{1}{3}x$. Then,

$$D = M_0\frac{x}{2} \times \left(\frac{5}{3}x - \frac{1}{3}x\right) \times \frac{1}{EI} = \frac{2M_0x^2}{3EI}. \quad . \quad . \quad . \quad (11)$$

The following is a numerical example in accordance with the condition illustrated in Fig. 46; and in which $l=240$ ins., $a=60$ ins., $(l-a)=180$ ins., $P=16{,}000$ lbs.

$$M=P\frac{l-a}{l}a=\frac{16{,}000\times180\times60}{240}=720{,}000 \text{ in.-lbs.};$$

$$M_0=-\frac{Pa(l^2-a^2)}{2l^2}=-\frac{16{,}000\times60(240^2-60^2)}{2\times240^2}=-450{,}000 \text{ in.-lbs.};$$

$$M_1=M-M_0\frac{a}{l}=720{,}000-\left(450{,}000\times\frac{60}{240}\right)=607{,}500 \text{ in.-lbs.};$$

$$x=\frac{M_0l(l-a)}{Ml+M_0(l-a)}=\frac{450{,}000\times240\times180}{(720{,}000\times240)+(450{,}000\times180)}=76.6 \text{ ins.};$$

$$R_2=\frac{M_0}{x}=\frac{450{,}000}{76.6}=5{,}874 \text{ lbs.}$$

$$R_1=P-R=16{,}000-5{,}874=10{,}126 \text{ lbs.}$$

Since the maximum moment, which is at the point of loading, $=607{,}500$ in.-lbs., then, for a permissible stress on the outer fibres of 16,000 lbs. per sq.in., the section modulus required $=\frac{607{,}500}{16{,}000}=37.97$. Thus a 12-inch I-beam at 35 lbs. per foot, for which $S=38.0$ and $I=228.3$, is the most economical section to employ. Then

$$D=\frac{2M_0x^2}{3EI}=\frac{2\times450{,}000\times76.6^2}{3\times29{,}000{,}000\times228.3}=0.266 \text{ in.}$$

A numerical example will now be considered in which the load is less than $2x$ from the fixed end, as illustrated in Fig. 47.

Here $l=240$ ins., $a=180$ ins., $(l-a)=60$ ins., $P=16{,}000$ lbs. Now

$$M=P\frac{l-a}{l}a=\frac{16{,}000\times60\times180}{240}=720{,}000 \text{ in.-lbs.};$$

$$M_0=-\frac{Pa(l^2-a^2)}{2l^2}=-\frac{16{,}000(240^2-180^2)}{2\times240^2}=-630{,}000 \text{ in.-lbs.};$$

$$M_1=M-M_0\frac{a}{l}=720{,}000-\left(630{,}000\times\frac{180}{240}\right)=247{,}500 \text{ in.-lbs.};$$

$$x=\frac{M_0l(l-a)}{Ml+M_0(l-a)}=\frac{630{,}000\times240\times60}{(720{,}000\times240)+(630{,}000\times60)}=43.08 \text{ ins.}$$

The maximum moment, which occurs at the fixed end, $=630,000$ in.-lbs.; and $\frac{630,000}{16,000}=39.37$, which is the section modulus required; thus a 12-inch I-beam at 40 lbs. per foot, for which $S=44.8$ and $I=268.9$, is the most economical section to use.

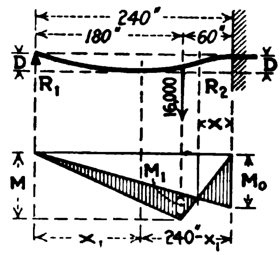


FIG. 47.

The distance x_1 of the point of maximum deflection from the supported end of the beam will now be determined by the condition that the moment of the moment area on one side of this point, about the adjacent end of the beam, shall be equal to the moment of the moment area on the other side of the point, about the other end of the beam.

The positive moment area $=247,500\times\frac{196.92}{2}=24,368,850$; and the distance of its centre of gravity from the supported end of the beam $=\frac{180+196.92}{3}=125.64$ ins.; while the distance of its centre of gravity from the fixed end of the beam $=240-125.64=114.36$ ins.

The negative moment area $=\frac{630,000\times43.08}{2}=13,570,200$; and the distance of its centre of gravity from the fixed end of the beam $=\frac{43.08}{3}=14.36$ ins.

The moment area on the length $x_1=\frac{247,500x_1^2}{180\times2}=\frac{1,375x_1^2}{2}$; and the distance of its centre of gravity from the supported end $=\frac{2}{3}x_1$, while that from the fixed end $=240-\frac{2}{3}x_1$. Then

$$\frac{1,375x_1^2}{2}\times\frac{2}{3}x_1=(24,368,850\times114.36)-\frac{1,375x_1^2}{2}(240-\tfrac{2}{3}x_1)$$
$$-(13,570,200\times14.36),$$

from which $\qquad x_1=125.33$ ins.

Now the moment area A on the length $x_1=\frac{1,375x_1^2}{2}=\frac{1,375\times125.33^2}{2}$ $=10,799,800$; and the distance X of its centre of gravity from the supported end $=\frac{2}{3}x_1=83.55$ ins.; then

$$D=\frac{AX}{EI}=\frac{10,799,800\times83.55}{29,000,000\times268.9}=0.1157 \text{ in.}$$

CASE IX. Beam fixed at one end and supported at the other, carrying a uniform load. Fig. 48.

The moment-diagram is a parabola of depth at the centre of span equal to $\frac{wl^2}{8}=M$. The negative bending moment M_0 is determined as in Case VIII, viz., the negative moment area (above the closing line), multiplied by the distance of its centre of gravity from the supported end of beam, must be equal to the positive moment area (below the closing line), multiplied by the distance of its centre of gravity from the same point; or, since the triangle of base l and altitude M_0 overlaps the parabolic segment, the area not shaded is common to both, and the moments of these two figures about the supported end of the beam must be equal to one another. The area of the triangle $=\frac{M_0 l}{2}$, and the distance of its centre of gravity from the supported end of the beam $=\frac{2}{3}l$. The area of the parabolic segment $=\frac{wl^2}{8}\cdot\frac{2}{3}l$ $=\frac{wl^3}{12}$, and the distance of its centre of gravity from the supported end of the beam $=\frac{1}{2}l$. Then

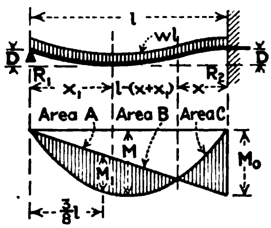


FIG. 48.

$$\frac{M_0 l}{2}\cdot\frac{2}{3}l=\frac{wl^3}{12}\cdot\frac{1}{2}l,$$

from which

$$M_0=\frac{wl^2}{8}=M. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The point of contraflexure is at the intersection of the closing line with the parabola, and the distance x of this point from the fixed end of the beam may be determined by equating the closing line with the parabola, thus:

$$\frac{wl^2}{8}\cdot\frac{l-x}{l}=\frac{wl^2}{8}-\frac{w}{2}\left(\frac{l}{2}-x\right)^2,$$

from which

$$x=\frac{1}{4}l. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The maximum positive moment will be found at the middle point of the positive moment area, or $\frac{3}{8}l$ from the supported end of the beam. Then

$$M_1=\frac{wl^2}{8}-\frac{3}{8}\cdot\frac{wl^2}{8}-\frac{w}{2}\left(\frac{l}{8}\right)^2=\frac{9}{128}wl^2. \quad . \quad . \quad . \quad . \quad (14)$$

The reactions may be determined by taking moments of the vertical forces about the fixed end of the beam, thus:

$$R_1l-\frac{wl^2}{2}=M_0=-\frac{wl^2}{8},$$

from which

$$R_1=\tfrac{3}{8}wl, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and therefore

$$R_2=\tfrac{5}{8}wl. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The distance x_1 of the point of maximum deflection from the supported end of the beam may be determined by the condition that the moment (about this end) of the area between it and the point of maximum deflection shall be equal to the algebraic sum of the moments (about the fixed end) of the areas between the point of maximum deflection and the fixed end. This condition may be realized from trial and error, first assuming a point of maximum deflection and computing the various moment areas and their centres of gravity by dividing the areas into elementary triangles and rectangles. Although this operation is quite feasible, it will be found very tedious; and it may be as well to accept the result obtained by the calculus, viz.:

$$x_1=\frac{1+\sqrt{33}}{16}l=0.4215l. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The moment area A on the length x_1 may now be calculated. It will be found to be equal to one-sixth of the rectangle which circumscribes the whole parabolic segment, $=\frac{1}{6}\cdot\frac{wl^3}{8}=\frac{1}{48}wl^3$; and the distance X of its centre of gravity from the supported end of the beam will be found equal to $0.26l$. Then

$$D=\frac{AX}{EI}=\frac{1}{48}wl^3\times 0.26l\times\frac{1}{EI}=\frac{0.005416wl^4}{EI}. \quad . \quad . \quad . \quad . \quad (18)$$

The same result may be obtained by taking moments about the fixed end of the beam, of the areas B and C. It will be found that these areas are exactly equal to one another, and are equal to $\frac{11}{96}$ of the rectangle circumscribing the parabolic segment, only one is positive and the other negative; thus $B=\frac{11}{96}\times\frac{wl^3}{8}=+\frac{11}{768}wl^3$ and $C=-\frac{11}{768}wl^3$. The distance X_1 of the centre of gravity of the area B from the fixed end of the beam$=0.4578l$, and the distance X_2 of the centre of gravity of the area C from the same point$=0.0795l$. Then

$$D=\frac{BX_1-CX_2}{EI}=\frac{11}{78}wl^3(0.4578l-0.0795l)\frac{1}{EI}=\frac{0.05416wl^4}{EI},$$

as before.

It should be noted that the positive moment area between the point of maximum deflection and the point of contraflexure is, in all cases, exactly equal to the negative moment area between the point of contraflexure and the fixed end of the beam.

CHAPTER IV

ART. 15. COLUMNS AND STRUTS

ALL columns, except those in which the length does not exceed about three times the least diameter, are subject to a bending stress in addition to a uniform compressive stress. The bending stress is due to the eccentricity of the load, and to the deflection caused thereby; for the load can never be applied at the exact centre of the column; the axis of the column will not be absolutely straight; nor will the material of the column be perfectly uniform throughout, owing to irregularities in rolling, or to initial stresses caused by straightening and rivetting.

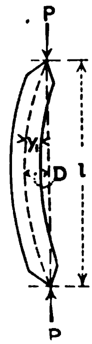


FIG. 49.

Fig. 49 represents a column free to turn at both ends. The load and its reaction, which are both represented by P, are applied at the extremities of the column and cause it to deflect as shown, the maximum deflection at the centre being represented by D. The uniform compressive stress per sq.in. p, Fig. 50, is equal to the load P divided by the area of the cross-section A; thus

$$p=\frac{P}{A}. \qquad (1)$$

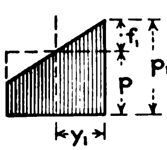


FIG. 50.

The maximum intensity of stress f_1 due to bending is equal to the moment at the middle point of the column, multiplied by the distance y_1 from the neutral axis to the extreme fibres of the concave side of the column, and divided by the moment of inertia of the section, thus:

$$f_1=\frac{My_1}{I}.$$

But, since $M=PD$, therefore

$$f_1=\frac{PDy_1}{I}. \qquad (2)$$

Now, assuming the curvature of the column to be circular (which is approximately correct), D may be determined as follows:

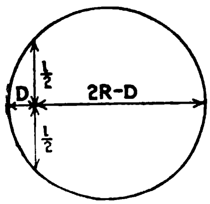


FIG. 51.

In Fig. 51, R=radius of curvature of the bent axis of column, in ins.,
l=length of column, in ins.,
D=deflection at middle point, in ins.

Then $$\left(\frac{l}{2}\right)^2=D(2R-D)=2DR \text{ (approx.)}.$$

$$\therefore \qquad D=\frac{l^2}{8R}. \qquad (3)$$

From Art. 14, equation (1), $\frac{y_1}{R}=\frac{f_1}{E}$; thus $R=\frac{Ey_1}{f_1}$; and, if this value of R be substituted in (3),

$$D=\frac{f_1}{8E}\cdot\frac{l^2}{y_1}=a\frac{l^2}{y_1},$$

where a is a constant $=\frac{f_1}{8E}$.

Substituting this value of D in (2),

$$f_1=\frac{P}{I}al^2=\frac{P}{A}a\frac{l^2}{r^2}=pa\frac{l^2}{r^2};$$

or $I=Ar^2$, and $\frac{P}{A}=p$. Then, the maximum stress per sq.in.,

$$p_1=p+f_1=p\left(1+a\frac{l^2}{r^2}\right)=\frac{P}{A}\left(1+a\frac{l^2}{r^2}\right).$$

If p_1 represent the elastic limit of the material in compression, which is about 42,000 lbs. per sq.in. for mild steel, then

$$P\text{, the breaking load, }=\frac{p_1A}{1+a\frac{l^2}{r^2}}. \qquad (4)$$

Equation (4) is Rankine's formula, in which

A = area of cross-section in sq.ins.;
p_1 = the elastic limit of the material in compression, in lbs. per sq.in.;
l = length of column, in ins.;
r = the least radius of gyration, in ins.;
a = a constant depending on the material, and determined experimentally by tests of actual columns.

If both ends of the column are fixed, as in Fig. 52, the load which it will carry before failure is the same as for a column of one-half its length, but free to turn at the ends; thus $\frac{l}{2}$ must be substituted for l and the formula becomes (for both ends square or fixed),

$$P=\frac{p_1 A}{1+\frac{a}{4}\frac{l^2}{r^2}} \qquad (5)$$

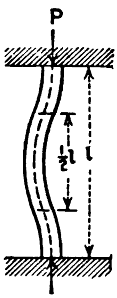


FIG. 52.

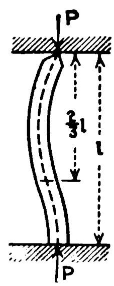


FIG. 53.

If one end of the column is fixed and the other end free to turn, as in Fig. 53, $\frac{2}{3}l$ must be substituted for l, and the formula becomes, (for one end square or fixed, and the other end free to turn),

$$P=\frac{p_1 A}{1+\frac{4}{9}a\frac{l^2}{r^2}} \qquad (6)$$

In practice, however, no column is perfectly free to turn at the ends, and it rarely happens that any column is absolutely fixed; thus

it becomes necessary to assume arbitrary values for a for the three cases instead of using the modified formulæ (5) and (6).

Many engineers advocate the use of one formula only both for columns with pin ends and for those with rivetted or fixed ends, assuming that in the latter case the accidental eccentricity of loading due to a possible unevenness of bearing or to slightly mismatched rivet holes is liable to induce bending moments fully as great as would occur in columns with pin ends. This appears to be a reasonable practice, which is here recommended, especially as many of the recent tests of full size columns give somewhat unfavorable results for those having a comparatively low ratio of $\frac{l}{r}$.

In addition to the uniform compressive stress in a column, and to the stress induced by flexure as a whole, there are many secondary stresses, some of which are as follows: (*a*) stress due to the flexure of the unsupported parts between lattice-bar connections; (*b*) stress due to the eccentricity of the end connections; (*c*) stress due to the eccentricity of the lattice-bar connections, for these bars seldom intersect on the centre of gravity line of the channels which they connect; (*d*) stress induced by the omission of tie-plates and lattice-bars at or near panel-points, which omission is sometimes unavoidable. These secondary stresses are usually independent of the length of a column or of the ratio $\frac{l}{r}$, and they should be provided for by making an allowance in the assumed working value of p_1 in Rankine's formula, equation (4).

Although the resistance of structural steel (in cubes) to crushing is approximately equal to its resistance to tension, the plates and shapes used in built columns will fail under a much smaller load, as indicated by recent tests of columns, the majority of which failed in detail rather than by flexure as a whole. In this connection, some interesting experiments on hollow cylindrical columns have been made by Mr. W. E. Lilly of Trinity College, Dublin, an account of which is published in "Engineering" for Jan. 10th, 1908, and reprinted in "The Engineering Digest" of March, 1908. Mr. Lilly has derived a formula for determining the proper working value of p_1 for circular sections based on the ratio $\frac{r}{t}$, where r is the radius of gyration and t the thickness of metal. Unfortunately, this formula is inapplicable to columns of other forms; but an extensive series of tests of columns

of various forms is now being conducted at the Watertown Arsenal, which should shed much light on this important matter.

In the meantime, however, for mild steel columns, 12,000 lbs. per sq.in. (or three-quarters of the permissible unit-stress in tension) is recommended for the working value of p_1, and $\frac{1}{18,000}$ for the coefficient a. Then

$$p = \frac{12,000}{1+\frac{l^2}{18,000r^2}}, \quad \dots \dots \dots \quad (7)$$

where $p=\frac{P}{A}$, the average allowable stress per sq.in. on the column.

The curve obtained from equation (7) is shown in Fig. 54; also

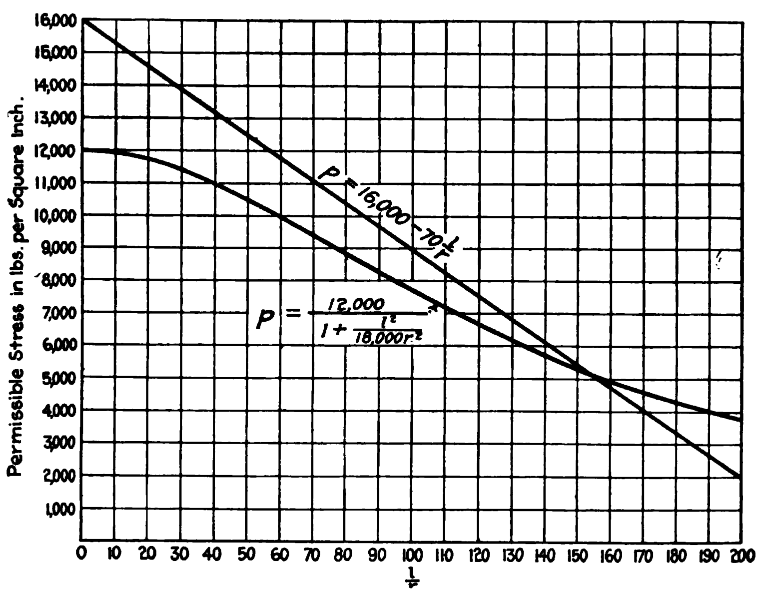


FIG. 54.

the straight line representing the equation, $p=16,000-70\frac{l}{r}$, which is the column formula adopted by the American Railway Engineering and Maintenance of Way Association. This latter formula undoubtedly gives values for the shorter columns which are excessive, as maintained by Mr. J. R. Worcester in a paper contributed in 1908 to the American Society of Civil Engineers entitled "Safe Stresses in Steel

Columns." This view was also held by the majority of the members who took part in the discussion of this paper.

The application of Rankine's formula to the design of columns will be greatly facilitated by employing Table I, which has been computed for values of $\frac{l}{r}$ varying from 10 to 209. The permissible unit stress for any given ratio of $\frac{l}{r}$ is found at once by inspection, thus: For a column 16 ft. long whose least radius of gyration = 4 ins., $\frac{l}{r}=\frac{16\times 12}{4}=48$; and the corresponding unit-stress found opposite this number = 10,630 lbs. per sq.in.

Forms of Columns in General Use. For roof trusses and similar structures two angles, back to back, are most frequently employed. One angle should never be used as a strut, except in cases where the stress is very small.

In buildings, single H-beams with wide flanges make excellent and economical columns for loads within their capacity. These beams are now rolled both in Europe and the United States. In Table II will be found the properties of H-beams rolled by Carnegie. For heavier loads, Z-bar columns, consisting of four Z-bars and a web-plate, thus: (with or without flange plates) are very suitable; but equally good columns may be formed of two channels back to back with an I-beam between them, thus: , or of three I-beams, thus: . Box sections, built of plates and angles, thus: are also highly efficient, and are suitable for very heavy loads; but they should be filled with cement grout to prevent corrosion.

In bridge trusses, the top chords and end posts are most frequently made of two channels and a cover-plate, thus: ; or of two web-plates, four angles and a cover-plate, thus: ; both of which are latticed on the bottom flanges. Sometimes the cover-plate is omitted from these sections, and they are latticed on both top and bottom

Table I

PERMISSIBLE UNIT STRESSES FOR MILD STEEL COLUMNS, IN ACCORDANCE WITH THE FORMULA

$$p=\frac{12{,}000}{1+\dfrac{l^2}{18{,}000r^2}},$$

in which p = average permissible stress in lbs. per sq.in.;
l = length of column in inches;
r = least radius of gyration in inches.

$\frac{l}{r}$	p	$\frac{l}{r}$	p	$\frac{l}{r}$	p	$\frac{l}{r}$	p	$\frac{l}{r}$	p
10	11,930	50	10,530	90	8,280	130	6,190	170	4,600
11	11,920	51	10,480	91	8,220	131	6,140	171	4,570
12	11,900	52	10,430	92	8,160	132	6,100	172	4,540
13	11,890	53	10,380	93	8,100	133	6,050	173	4,510
14	11,870	54	10,330	94	8,050	134	6,010	174	4,470
15	11,850	55	10,270	95	7,990	135	5,960	175	4,440
16	11,830	56	10,220	96	7,930	136	5,920	176	4,410
17	11,820	57	10,170	97	7,880	137	5,870	177	4,380
18	11,790	58	10,110	98	7,820	138	5,830	178	4,350
19	11,760	59	10,060	99	7,770	139	5,780	179	4,320
20	11,740	60	10,000	100	7,710	140	5,740	180	4,290
21	11,710	61	9,940	101	7,660	141	5,700	181	4,260
22	11,690	62	9,890	102	7,600	142	5,660	182	4,230
23	11,660	63	9,840	103	7,550	143	5,620	183	4,200
24	11,630	64	9,780	104	7,500	144	5,580	184	4,170
25	11,600	65	9,720	105	7,440	145	5,530	185	4,140
26	11,570	66	9,660	106	7,390	146	5,490	186	4,110
27	11,530	67	9,600	107	7,340	147	5,450	187	4,080
28	11,500	68	9,540	108	7,280	148	5,410	188	4,050
29	11,460	69	9,490	109	7,230	149	5,370	189	4,020
30	11,430	70	9,430	110	7,180	150	5,330	190	3,990
31	11,390	71	9,370	111	7,120	151	5,290	191	3,960
32	11,350	72	9,320	112	7,070	152	5,250	192	3,930
33	11,320	73	9,260	113	7,020	153	5,220	193	3,910
34	11,280	74	9,200	114	6,970	154	5,180	194	3,880
35	11,240	75	9,140	115	6,920	155	5,140	195	3,860
36	11,190	76	9,080	116	6,870	156	5,100	196	3,830
37	11,150	77	9,020	117	6,820	157	5,060	197	3,800
38	11,110	78	8,960	118	6,770	158	5,030	198	3,780
39	11,060	79	8,900	119	6,720	159	4,990	199	3,750
40	11,020	80	8,850	120	6,670	160	4,950	200	3,720
41	10,970	81	8,800	121	6,620	161	4,920	201	3,700
42	10,920	82	8,740	122	6,570	162	4,880	202	3,670
43	10,880	83	8,680	123	6,520	163	4,850	203	3,650
44	10,830	84	8,620	124	6,470	164	4,810	204	3,620
45	10,780	85	8,560	125	6,420	165	4,770	205	3,600
46	10,730	86	8,500	126	6,380	166	4,740	206	3,570
47	10,680	87	8,460	127	6,330	167	4,710	207	3,550
48	10,630	88	8,400	128	6,280	168	4,670	208	3,530
49	10,580	89	8,340	129	6,240	169	4,640	209	3,500

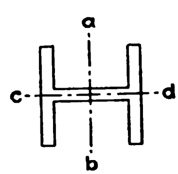
flanges. The compression web members are usually constructed of two channels, thus:] [, latticed on both sides; or of four angles and a web-plate, thus: ⊢⊣. In some cases, latticing is used instead of the web-plate; but it is not as efficient, particularly for a vertical post having a floor beam connected to one flange. This section is also very suitable for top chords and end posts, as well as for bottom chords; but it has not been used very extensively in the former capacity, probably owing to its general appearance.

In order to guard against failure by crinkling, relatively thin material should be avoided. The thickness of web-plates should not be less than one-thirtieth of the distance between their connections; the thickness of cover-plates should not be less than one-fortieth of the distance between rivet lines; and the thickness of outstanding flanges should not be less than one-twelfth of their width.

Table II

PROPERTIES OF CARNEGIE H-BEAMS

Section, Index.	Size, Inches.	Area, Sq.ins.	Weight in Pounds per Foot.	Moments of Inertia.		Radii of Gyration.	
				Axis *ab*	Axis *cd*	Axis *ab*	Axis *cd*
H 1	4×4	3.99	13.6	11.06	4.16	1.66	1.02
H 2	5×5	5.46	18.7	24.55	9.02	2.12	1.28
H 3	6×6	7.00	23.8	46.56	16.75	2.58	1.55
H 4	8×8	10.17	34.6	121.51	41.02	3.46	2.01



Art. 16. The Latticing of Compression Members

Until quite recently, very little consideration has been given to the stresses induced in the lattice-bars of compression members; and it has been generally customary to proportion these bars in accordance with rules derived from years of practical experience. This method has undoubtedly given satisfactory results when applied to cases within the range of ordinary practice; but, when an exceptional case arises it is entirely inadequate. Hence the necessity of an intelligent understanding of the duties performed by lattice-bars, and of being able to compute (approximately, at least) the maximum stresses they are liable to sustain. This subject has already been treated of in another work by the author, entitled "The Design of Typical Steel Railway Bridges," but it will be dealt with here in a somewhat different manner, which is thought to be more satisfactory.

The principal duty of the latticing in an ordinary compression member, where there are no intentionally eccentric loads, is to provide for the shearing stresses induced by the flexure of the column, referred to in the previous article. In addition to this, the lattice-bars should be stiff enough to prevent their being bent during the manufacture and transportation of the column, and to provide for the shear due to its weight when lying horizontally and supported at both ends.

The shear due to flexure may be approximated by considering the column as a beam, supported at both ends and carrying a uniform load which would cause a maximum fibre-stress at the middle point equal to $f_1 = p_1 - p$, Fig. 50.

A formula for the maximum end shear will now be developed, using the following notation:

l = length of column, in inches;
A = area of cross-section of column, in sq.ins.;
I = moment of inertia about axis perpendicular to plane of latticing;
y_1 = distance, in inches, from this axis to outer fibres;
r = radius of gyration, in inches, about the same axis;
p_1 = maximum fibre stress, in lbs. per sq.in.;
p = average permissible stress, in lbs. per sq.in.;
f_1 = maximum fibre stress, due to flexure only, in lbs. per sq.in. $= p_1 - p$;

w = uniform load, in lbs. per lin.in.;
M = bending moment in in.-lbs. at middle point of column;
Q = end shear, in lbs., perpendicular to longitudinal axis of column.

Now

$$M=\frac{wl^2}{8}=\frac{I}{y_1}f_1=\frac{Ar^2f_1}{y_1}, \quad \ldots \ldots \quad (1)$$

and

$$Q=\frac{wl}{2}; \quad \ldots \ldots \quad (2)$$

hence

$$\frac{M}{Q}=\frac{wl^2}{8}\cdot\frac{2}{wl}=\frac{l}{4}; \quad \ldots \ldots \quad (3)$$

finally

$$Q=\frac{4M}{l}=\frac{4Ar^2f_1}{y_1l}. \quad \ldots \ldots \quad (4)$$

When the straight-line formula $\left(p=16{,}000-70\frac{l}{r}\right)$ of the American Railway Engineering and Maintenance of Way Association is used in proportioning the column, equation (4) becomes

$$Q=\frac{4Ar^2}{y_1l}\cdot\frac{70l}{r}=280A\frac{r}{y_1}. \quad \ldots \ldots \quad (5)$$

The stress in the end lattice-bars will then be equal to the shear, by equations (4) or (5), multiplied by the secant of the angle between the lattice-bars and a line perpendicular to the longitudinal axis of the column. The lattice-bars should then be proportioned by the column formula, equation (7), Art. 15. The width of lattice-bars should not be less than two and one-half times the diameter of the rivets used therein; and the thickness of single lattice-bars should not be less than one-fortieth of their length c. to c. of rivets; while double lattice-bars, rivetted at their intersections, should not be of less thickness than one-sixtieth of their length c. to c. of end rivets. Generally, single latticing should make an angle of 60° with the longitudinal axis of the column; and double latticing, an angle of 45°; but in no case should the ratio $\frac{l}{r}$, where l is the length between lattice-bar connections

on one channel of a column, and r the least radius of gyration of the channel, be greater than the ratio $\frac{l}{r}$ for the column as a whole. The size of lattice-bars and their spacing should be uniform throughout the length of the column, and they should be connected directly to the end tie-plates without any intervening space.

The above discussion will now be illustrated by a numerical example, viz., a column composed of two 15-in. at 33 lbs. channels, 9.5 ins. back to back, and 30 ft. long. The sectional area of the column $=19.8$ sq.ins.; its radius of gyration r, about the axis perpendicular to the planes of latticing, $=5.62$ ins.; and the distance y_1 from this axis to the outer fibres $=8.25$ ins. Now $l=30\times 12=360$ ins., and $\frac{l}{r}=\frac{360}{5.62}=64$. Therefore, from Table I, $p=9,780$; then $f_1=p_1-p=12,000-9,780=3,220$ lbs. per sq.in.; and $Q=\frac{4Ar^2f_1}{y_1l}=\frac{4\times 19.8\times 5.62^2\times 3,220}{8.25\times 360}$ $2,700$ lbs. Since there are two planes of latticing, the shear for one lattice-bar at end of column $=2,700\times\frac{1}{2}=1,350$ lbs.; and, if the latticing be at $60°$ with the longitudinal axis of the column, this shear should be multiplied by the secant of $30°$ in order to obtain the stress in the bar, thus: $1,350\times 1.155=1,560$ lbs. The length c. to c. of end rivets of the lattice-bars $=15.6$ ins.; and therefore, by the general rule, their thickness thould be equal to one-fortieth of their length, $=\frac{3}{8}$ in. If $\frac{7}{8}$ in. rivets be used, the width of the bars should be $2\frac{1}{4}$ ins. Then $2\frac{1}{4}\times\frac{3}{8}$ in. bars will be suitable provided they are sufficiently strong as struts. The least radius of gyration of the bar $=0.11$ in.; then $\frac{l}{r}=\frac{15.6}{0.11}=142$, which, by Table I, corresponds to a permissible unit stress of 5,160 lbs. per sq.in.; and the value of the bar in compression $=2\frac{1}{4}\times\frac{3}{8}\times 5,160$ $=4,350$ lbs., which is considerably greater than the stress in it due to flexure.

CHAPTER V

ART. 17. LOADS CARRIED BY VARIOUS STRUCTURES

BUILDINGS and bridges, generally, are designed for three classes of loads, viz., *dead-load, live-load* and *wind-load.* The dead-load consists of the weight of the structure itself, including floors, roofs, walls, and all other permanent construction. The live-load, which is variable, consists of the weight of people, movable furniture, horses, vehicles, snow, etc. The wind-load is the force of the wind acting on the exposed surfaces of a structure, or on the surface of any vehicle or other body which may be passing over or supported by the structure. Since the weight of a structure cannot be known definitely until it has been designed, it is necessary to assume some approximate weight in order to calculate the dead-load stresses. If, on completion of the design, it is found that the assumed weight is either too great or too small, it may then be corrected and the stresses re-calculated.

In the following table will be found the weights of the principal materials used in the construction of buildings and bridges.

TABLE III

WEIGHTS OF MATERIALS, ETC.

Name.	Lbs. per cu.ft.	Name.	Lbs. per cu.ft.
Brickwork, common	100 to 120	Stone, limestone	160
Concrete, stone	130 to 150	" sandstone	145
" cinder	70	" slate	175
Coal, anthracite, loose	54	Slag	40
" bituminous, loose	49	Sand, clay, and earth (dry)	100
Gravel	120	" " " " (wet)	120
Iron, cast	450	Snow (freshly fallen)	10
" wrought	480	" (wet)	50
Lumber, spruce and hemlock	30	Steel	490
" white and red pine	35	Water, pure, at 60° F.	62½
" Douglas fir	40		Lbs. per sq.ft. (laid).
" yellow pine	45		
" white oak	50	Corrugated iron, No. 20 gauge	2.06
Masonry, ashlar	140 to 160	" " No. 22 gauge	1.75
" rubble	130 to 150	" " No. 24 gauge	1.43
Mortar	100	" " No. 26 gauge	1.13
Paving brick	150	Roof slates	8.00 to 10.00
" asphaltum	100	Roofing, tar and gravel	1.25
Stone, granite and marble	170	Plastering for walls and ceilings	10.00

Approximate Weight of Roof Trusses. For the purpose of calculating the stresses, it will be quite accurate enough to assume that the weight of the trusses is equal to 5 lbs. per sq.ft. of area covered. Thus, if the span be 60 ft. and the distance between trusses 15 ft., then the approximate weight of one truss will be $60 \times 15 \times 5 = 4{,}500$ lbs. When steel purlins are used, their weight may also be assumed at 5 lbs. per sq.ft. of area covered.

Snow-Load on Roofs. For temperate climates a snow-load of 25 lbs. per sq.ft. of roof surface should be assumed for all slopes up to 20°. For greater slopes this load may be decreased 1 lb. for each additional degree up to 45°, when no snow-load need be considered. In severe climates, or in sheltered places where the snow is liable to fill in, the above snow-load should be increased in accordance with the actual conditions.

Wind-Load on Sides and Roofs of Buildings. On all vertical surfaces the wind load is usually taken at 30 lbs. per sq.ft., acting in a horizontal direction. On sloping surfaces it may be assumed to act in a direction perpendicular to the slope, and its intensity taken from Table IV, which has been computed in accordance with Unwin's formula.

TABLE IV

WIND PRESSURES ON ROOFS

Angle of Roof with Horizontal.	Normal Pressure in lbs. per sq.ft.	Angle of Roof with Horizontal.	Normal Pressure in lbs. per sq.ft.
5°	3.8	40°	24.9
10°	7.2	50°	28.5
20°	13.5	60 to 90°	30.0
30°	19.8		

Floor Loads for Buildings. In addition to the weight of flooring and supporting beams, the floors of buildings should be designed to carry the maximum live-loads to which they are liable to be subjected; but in no case should the assumed live-loads be less than those given in Table IV*a*.

TABLE IV*a*

MINIMUM LIVE-LOADS FOR FLOORS OF BUILDINGS

	Lbs. per sq. ft.
Dwellings, hotels, etc.	50
Upper stories of office buildings	60
Churches, schoolrooms, theatre galleries, also flat roofs liable to be crowded by people	75
Ground floors of office buildings, main floors of theatres, dancing halls, etc.	100
Warehouses and factories	150 to 600

Crane Loads. When structures are required to support travelling cranes or conveyors of any sort, provision should be made for the effect of impact and vibrations induced thereby. This may be accomplished by adding about 25 per cent to the weight of these moving parts and their loads.

The top flanges of crane runway girders should be designed to resist, in addition to the vertical load, a lateral force which may be developed in the attempt to lift a load obliquely. This lateral force may be taken at 20 per cent of the maximum load to be lifted, and is assumed to be distributed equally to the four wheels of the crane.

The Dead-Load for Highway Bridges is usually divided into two parts, viz.: *a*, the weight of timber or concrete floor, *b*, the weight of steel. The weight of the floor may be computed from data given in Table III. For an ordinary country bridge of 16 ft. roadway, with wooden stringers, flooring and hand-rails, the weight of steel may be determined approximately by the formula

$$w_s = 2.5L + 75, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which w_s = weight of steel per lin.ft. of bridge,
L = length of span in feet.

For a similar structure of greater or less width, the weight of steel will be proportionately greater or less; and, if the bridge have steel stringers and hand-railings, the weight of these items should be calculated and added to the result given by formula (1).

Live-Load for Highway Bridges. For the trusses of an ordinary bridge on a country road, a live-load of 75 lbs. per sq.ft. of roadway and sidewalks (if any) is ample; but the floor system and its immediate supports should be designed for a load of not less than 100 lbs. per sq.ft.; or for a load of 16,000 lbs. equally distributed on four wheels, 6 ft. apart transversely and 8 ft. c. to c. of axles.

For city bridges, or for bridges designed to carry electric railways, the live-load should be assumed in accordance with the actual conditions, allowing for the heaviest cars, road rollers, etc.; but in no case should it be taken at less than 100 lbs. per sq.ft. of roadway and sidewalks, whether for trusses or floor system.

Impact for Highway Bridges. To provide for the effect of impact and vibration due to the live-load, there should be added to the calculated stresses for dead- and live-loads an impact stress which may be computed by the formula

$$Imp. = \frac{range^2}{2\ max.}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which *imp.* = the impact stress caused by the live-load;
range = the range or numerical sum of the live-load stresses;
max. = the maximum stress due to the dead- and live-loads which can occur at one time.

When the live-load stress in a member is always of the same kind (whether tension or compression), then the maximum live-load stress represents the *range;* but, when a member is subject to alternate live-load stresses of tension and compression, then the numerical sum of these alternate stresses represents the *range.*

Wind-Loads on Highway Bridges. The top laterals of deck bridges and the bottom laterals of through bridges are usually proportioned for a horizontal wind force of 300 lbs. per lin.ft., considered as a moving load; and the bottom laterals of deck bridges and the top laterals of through bridges, for a horizontal wind force of 150 lbs. per lin.ft., also considered as a moving load. For through pony-truss bridges having bottom laterals only, the horizontal wind force may be taken at 450 lbs. per lin.ft., considered as a moving load acting in the plane of the laterals.

The loads for railway bridges will not be considered in this place, as the subject is treated quite fully in "The Design of Typical Steel Railway Bridges," previously referred to.

ART. 18. PERMISSIBLE UNIT-STRESSES

MILD STEEL

	Lbs. per sq.in.
Tension on net section	16,000
Compression on gross section	$\dfrac{12{,}000}{1+\dfrac{l^2}{18{,}000r^2}}$
Shearing: Shop rivets and pins	12,000
Field-driven rivets and bolts	9,000
Webs of beams and plate-girders, gross section.	10,000
Bearing: Shop rivets and pins	24,000
Field-driven rivets and bolts	18,000
Bending: On outer fibres of rolled shapes and built girders, net section	16,000
On outer fibres of pins	24,000

Bearing on expansion rollers, per lin.in., $1{,}200\sqrt{\text{diam. in ins.}}$

BEARING ON MASONRY

	Lbs. per sq.in.
Granite	500
Sound limestone and Portland cement concrete	400
Sandstone	300
Hard-burned brick, laid in Portland cement mortar	200
Common brick laid in lime mortar	100

BEARING ON SOILS

	Tons per sq.ft.
Soft clay	1
Ordinary clay and dry sand mixed with clay	2
Dry sand and dry clay	3
Hard clay and firm, coarse sand	4
Firm, coarse sand and gravel	6
Rock	25

TIMBER: PERMISSIBLE STRESSES IN POUNDS PER SQUARE INCH

Kind of Timber.	Bending on Outer Fibres.	End Bearing.	Columns under 10 Diameters	Bearing across Grain.	Shearing with Grain.
White oak	1500	1500	1200	600	300
Long-leaf yellow pine	1800	1800	1200	400	250
Douglas fir	1800	1800	1200	400	250
White and red pine	1200	1200	800	250	150
Spruce	1200	1200	800	250	150
Hemlock	1000	1000	700	200	100

The permissible unit-stresses for long timber columns may be determined by the following empirical formula

$$p=C+150-15\frac{l}{d}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which p = permissible stress per sq.in. on columns when $\frac{l}{d}>10$;

C = permissible stress per sq.in. on columns when $\frac{l}{d}<10$;

l = length of column in inches;

d = least side of column in inches.

Wind and Other Stresses Combined. For the combination of the maximum stresses produced by the wind-load with those due to the dead-load, live-load and impact, it is customary to permit of unit stresses 25 per cent higher than those given in this article; but the sections obtained thereby should not be less than required if the wind-load stresses be neglected. The reason for allowing the higher unit-stresses with the combination of stresses just mentioned is on account of its infrequency.

ART. 19. RIVETS AND RIVETTING

The sizes of rivets used in structural steelwork are $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$ and 1 in. Those in most general use are $\frac{3}{4}$ and $\frac{7}{8}$ in. The smaller ones are used only in very light members to avoid cutting out too much of the sectional area. Rivets larger than $\frac{7}{8}$ in. are difficult to drive, and they are only used in cases where it is impossible to get in enough of a smaller size, owing to lack of available space.

Spacing of Rivets. The distance centre to centre of rivets should not be less than three times their diameter. In compression members, rivets should not be farther apart, in line of stress, than sixteen times the thickness of the outside plates, nor should this spacing exceed six inches.. The distance from the centre of a rivet to the end of a sheared member should not be less than one and one-half times the diameter of the rivet, or generally, $1\frac{1}{4}$ ins. for $\frac{3}{4}$-in. rivets and $1\frac{1}{2}$ ins. for $\frac{7}{8}$-in. rivets.

Size of Rivet Holes. Ordinarily holes are punched $\frac{1}{16}$ in. larger than the rivets; but, in more particular work, they are punched $\frac{1}{8}$ in. smaller and, after assembling of the parts to be riveted, are reamed to $\frac{1}{16}$ in. larger than the rivets. Holes in metal of greater thickness than the diameter of the rivets are usually drilled, as they are difficult to punch and because the punching of comparatively thick material is considered to be too injurious to it. In the determination of the net sectional area of tension members, it is customary to make allowance for holes of $\frac{1}{8}$ in. greater diameter than that of the rivets before driving, or $\frac{1}{16}$ in. greater than the actual size of the holes. This extra $\frac{1}{16}$ in. is to compensate for the injury to the surrounding metal due to punching. In compression members, no allowance is made for rivet holes, the assumption being that the rivets entirely fill the holes and thus make up the loss of sectional area.

Strength of Rivets. Rivets may fail by shearing, by crushing (or bearing) or by tension on the heads. Generally, it is not considered good practice to use rivets in tension, as their tensile strength is somewhat uncertain owing to initial stresses from cooling; but it is sometimes unavoidable to use them in this manner, in which cases the tension should not exceed 8,000 lbs. per sq.in.

Shearing and Bearing Values of Rivets. The permissible shears on shop- and field-driven rivets, as given in the previous article, are respectively 12,000 and 9,000 lbs. per sq.in. Now the shearing value of a rivet is equal to the area of its cross-section, in sq.ins., multiplied by the permissible shear per sq.in. Thus, the shearing value of a $\frac{3}{4}$-in. rivet at 12,000 lbs. per sq.in. $=0.4418$ sq.in. $\times 12{,}000 = 5{,}300$ lbs.; and, at 9,000 lbs. per sq.in. $=0.4418 \times 9{,}000 = 3{,}975$ lbs.

The permissible bearings on shop- and field-driven rivets are given at 24,000 and 18,000 lbs. per sq.in. The bearing value of a rivet is equal to the diameter of the rivet multiplied by the thickness of metal on which it bears (both in ins.) multiplied by the permissible bearing per sq.in. Thus, the bearing value of a $\frac{3}{4}$-in. rivet on a $\frac{3}{8}$-in. plate, at 24,000 lbs. per sq.in. $=\frac{3}{4} \times \frac{3}{8} \times 24{,}000 = 6{,}750$ lbs.; and, at 18,000 lbs. per sq.in., $=\frac{3}{4} \times \frac{3}{8} \times 18{,}000 = 5{,}060$ lbs.

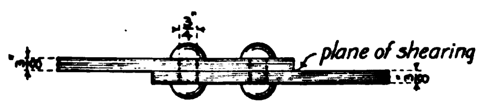


FIG. 55.

Rivets may be either in single or double shear. Fig. 55 represents a joint where the rivets are in single shear: that is to say, the joint could fail by the rivets shearing in one plane only, viz., that between the two members joined. Fig. 56 represents a joint where the rivets are in double shear. In this case the rivets would have to be sheared in two planes, as shown, before the joint could fail in that manner, and they would thus have twice the shearing value of the rivets in Fig. 55. In most cases, however, when rivets are in double shear their bearing value is less than twice their shearing value, when the bearing value determines the strength of the joint. In Fig. 55 each rivet is good for 5,300 lbs. in shear and 6,750 lbs. in bearing (if

shop rivets); and thus the shearing value governs. In Fig. 56 each rivet is good for $5{,}300 \times 2 = 10{,}600$ lbs. in shear and 6,750 lbs. in bearing; so the bearing value governs. If the centre member in Fig. 56 were $\frac{1}{2}$ in. thick the bearing value of the rivets would then be $\frac{3}{4} \times \frac{1}{2} \times 24{,}000 = 9{,}000$ lbs. each, which is still less than their shearing value. If this member were $\frac{5}{8}$ in. thick, the bearing value would be $\frac{3}{4} \times \frac{5}{8} \times 24{,}000$

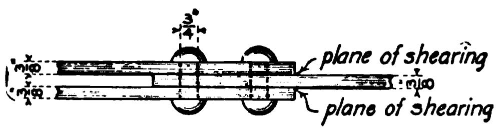


FIG. 56.

$= 11{,}260$ lbs.; therefore, the double shearing value of the rivets would determine the strength of the joint. In designing a rivetted connection, great care must be taken always to use the least value a rivet can have under the circumstances, whether in single shear, double shear, or bearing.

Table V gives the shearing and bearing values of shop- or power-driven rivets, computed for a permissible shear of 12,000 lbs. per sq.in., and a permissible bearing of 24,000 lbs. per sq.in.

Table VI gives the shearing and bearing values of field- or hand-driven rivets, computed for a permissible shear of 9,000 lbs. per sq.in., and a permissible bearing of 18,000 lbs. per sq.in.

In Tables V and VI, all bearing values above or to the right of the upper heavy lines are greater than double shear. Values between the upper and lower heavy lines are less than double, and greater than single shear. Values below and to the left of lower heavy lines are less than single shear.

TABLE V.—SHEARING AND BEARING VALUES FOR SHOP-DRIVEN RIVETS, IN POUNDS

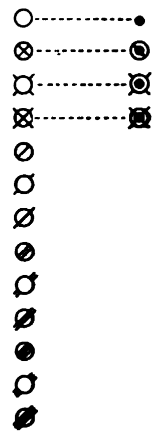
Diameters of Rivets in inches.	Single Shear at 12.000 lbs. per sq.in.	Bearing Values for Different Thicknesses of Plate at 24,000 lbs. per square inch.										
		1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	15/16	7/8
3/8	1,320	2,260	2,820	3,380								
1/2	2,360	3,000	3,760	4,500	5,260	6,000						
5/8	3,680	3,760	4,680	5,620	6,560	7,500	8,440					
3/4	5,300	4,500	5,620	6,760	7,980	9,000	10,320	11,260	12,380			
7/8	7,220	5,260	6,560	7,880	9,180	10,500	11,820	13,120	14,440	15,760	17,060	
1	9,430	6,000	7,500	9,000	10,500	12,000	13,500	15,000	16,500	18,000	19,500	21,000

TABLE VI.—SHEARING AND BEARING VALUES FOR FIELD-DRIVEN RIVETS, IN POUNDS

Diameters of Rivets in inches.	Single Shear at 9000 lbs. per sq.in.	Bearing Values for Different Thicknesses of Plate at 18,000 lbs. per square inch.										
		1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	13/16	7/8
3/8	990	1,695	2,115	2,035								
1/2	1,770	2,250	2,820	3,375	3,945	4,500						
5/8	2,760	2,820	3,510	4,215	4,920	5,625	6,330					
3/4	3,975	3,375	4,215	5,070	5,910	6,750	7,740	8,445	9,285			
7/8	5,415	3,945	4,920	5,910	6,885	7,875	8,865	9,840	10,830	11,820	12,795	
1	7,065	4,500	5,625	6,750	7,875	9,000	10,125	11,250	12,375	13,500	14,625	15,750

CONVENTIONAL SIGNS FOR RIVETTING, IN GENERAL USE IN THE UNITED STATES AND CANADA

	Shop Rivets.	Field Rivets.
Two full heads		
Countersunk far side and chipped		
Countersunk near side and chipped		
Countersunk both sides and chipped		
Countersunk far side but not chipped		
Countersunk near side but not chipped		
Countersunk both sides but not chipped		
Flattened to $\frac{1}{4}$ in. far side		
Flattened to $\frac{1}{4}$ in. near side		
Flattened to $\frac{1}{4}$ in. both sides		
Flattened to $\frac{3}{8}$ in. far side		
Flattened to $\frac{3}{8}$ in. near side		
Flattened to $\frac{3}{8}$ in. both sides		



CHAPTER VI

Example in Office Building Construction

Fig. 57 represents the steel framework of a small office building. The floors and roof are assumed to be of reinforced concrete construction; and the outer walls of brick, 12 ins. thick, carried on steel girders at each floor.

The loads for the first, second, and third floors, as well as the roof, will be taken as follows:

Cinder concrete, 4 ins. thick	25	
Plaster ceiling......................	10	
Wooden floor	5	
Steel floorbeams and stringers.	5	
	—	45
Live-load..........................		60
Total load per square foot		105 lbs.

Loads for ground floor:

Cinder concrete, 6 ins. thick	35	
Plaster ceiling......................	10	
Tile floor 1 in. thick	15	
Steel floorbeams and stringers.........	5	
	—	65
Live-load..........................		100
Total load per square foot		165 lbs.

The weight of the 12-inch curtain walls will be assumed at 120 lbs. per sq.ft. of surface.

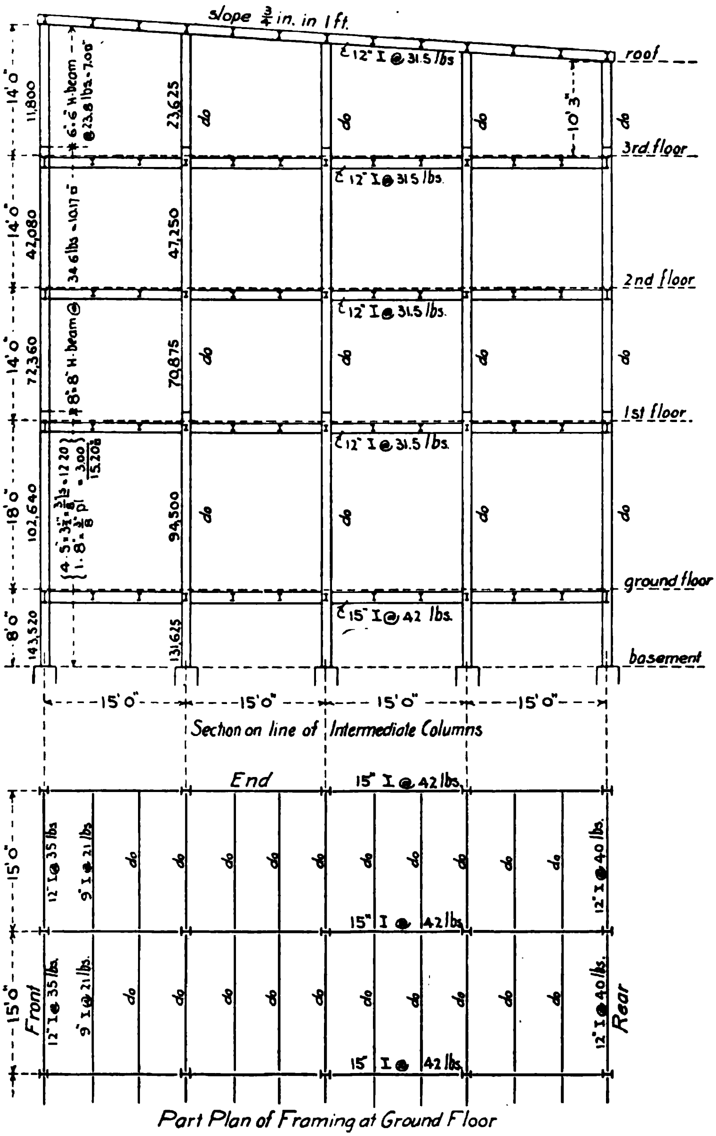

slope ¾ in. in 1 ft.
roof
3rd floor
2nd floor
1st floor
ground floor
basement
12" I @ 31.5 lbs
12" I @ 31.5 lbs.
12" I @ 31.5 lbs.
12" I @ 31.5 lbs.
15" I @ 42 lbs.
14'0"
14'0"
14'0"
18'0"
8'0"
11,800
42,080
72,360
102,640
143,520
23,625
47,250
70,875
94,500
131,625
10'3"
15'0"
15'0"
15'0"
15'0"
Section on line of Intermediate Columns
End
15" I @ 42 lbs.
12" I @ 35 lbs.
9" I @ 21 lbs.
12" I @ 40 lbs.
Front
Rear
Part Plan of Framing at Ground Floor

FIG. 57.

Stringers for Upper Floors and Roof. The span of the stringers is 15 ft., and they are 5 ft. apart c. to c. Then the total distributed load on each stringer $=15\times5\times105=7,875$ lbs.; the bending moment at the centre $=\frac{7,875\times15}{8}=14,760$ ft.-lbs. $=177,120$ in.-lbs; and the section modulus required $=177,120\div16,000=11.07$. Now, turning to the table of properties of standard I-beams in the hand book of any, of the rolling mills companies, it will be found that an 8-in. I-beam weighing 18 lbs. per foot, and having a section modulus of 14.2, is suitable for this place.

Floorbeams for Upper Floors and Roof. The span, used in computing the bending moment of the floorbeams, is the distance c. to c. of columns, viz., 15 ft. The loads, which are concentrated 5 ft. from either end, are each equal to the total load on one stringer, $=7,875$ lbs. The bending moment is a maximum at the stringer connections, and, in this particular case, is uniform or constant between them. The reaction at either end is equal to one of the concentrations, and the maximum moment is equal to the reaction multiplied by its distance from the near load, $=7,875\times5=39,375$ ft.-lbs. $=472,500$ in.-lbs. Then $S=472,500\div16,000=29.5$. For this case, a 12-in. I-beam at 31.5 lbs. per ft. and for which $S=36$, is suitable.

Stringers for Ground Floor. The span and distance apart of these stringers are the same as for those of the upper floors. The total distributed load on each stringer $=15\times5\times165=12,375$ lbs.; and the bending moment at the centre $=\frac{12,375\times15}{8}=23,200$ ft.-lbs $=278,400$ in.-lbs. Then $S=278,400\div16,000=17.4$. Here, a 9-in. I-beam at 21 lbs. per foot and for which $S=18.9$, is the beam to use.

Floorbeams for Ground Floor. The span and manner of loading are the same as for the upper floors. The concentrations are equal to the stringer loads $=12,375$ lbs.; and the maximum bending moment $=12,375\times5=61,875$ ft.-lbs. $=742,500$ in.-lbs. Then $S=742,500\div16,000=46.4$. For this case a 15-in. I-beam at 42 lbs. per foot for which $S=58.9$, is the most economical section to employ.

Front and Rear Wall Beams at First, Second, and Third Floors. If the walls were solid throughout, without window openings, the greatest load inducing bending moments on the supporting beams would be represented by a triangle of base equal to the span and altitude at the centre equal to about one-half the span, as indicated by the dotted lines in Fig. 58 (*a*), weighing 120 lbs. per sq.ft. of area;

for the remainder of the wall would be found to support itself by arch action.

Fig. 58(*a*) represents one panel of the wall, having two window openings 3 ft. 6 ins. by 8 ft. In this case the shaded area may be taken as the load in computing the bending moment. If, however, there were but one window in the centre of the panel it would be justifiable to assume that only the portion of the wall below the window

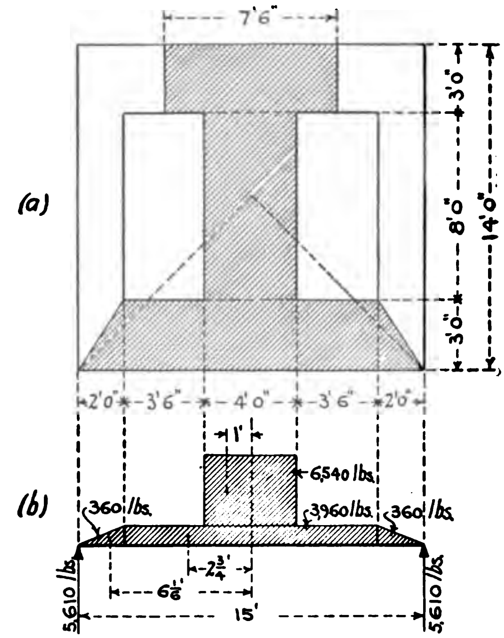


FIG. 58.

sill caused bending moments in the beam, as the remainder would evidently be self-supporting.

Fig. 58(*b*) represents the assumed wall loads in the present example. The lower rectangle on the central 11 ft. is equal to the weight of the wall below the window sills $= 11$ ft. $\times 3$ ft. $\times 120$ lbs. $= 3{,}960$ lbs. The triangular areas at the ends are each equal to the weight of the shaded part of the wall between either end of beam and the edge of the adjacent window, $= \frac{3 \times 2}{2} \times 120 = 360$ lbs. The upper rectangle is equal to the weight of the wall between the windows, as well as of the central half

of the wall above the windows, for the end portions are assumed to be supported by the side piers. Then, the weight of the upper rectangle $=(4\times 8\times 120)+(3\times 7.5\times 120)=6{,}540$ lbs. The reaction for these loads at either end of the beam will be equal to $\frac{3{,}960+6{,}540}{2}+360=5{,}610$ lbs. The bending moment at the centre will be equal to the reaction at either end multiplied by its distance from this point, minus the loads on either side of the centre multiplied by the distances of their several centres of gravity from the same point, thus:

$$
\begin{array}{lrr}
5{,}610\times 7\tfrac{1}{2}= & & +42{,}075 \\
-\frac{6{,}540}{2}\times 1 = & -3{,}270 & \\
-\frac{3{,}960}{2}\times 2\tfrac{3}{4} = & -5{,}445 & \\
-\ 360\times 6\tfrac{1}{6} = & -2{,}220 & -10{,}935 \\
\hline
\end{array}
$$

Moment at centre of beam from wall load $=+31{,}140$ ft.-lbs.

In addition to the wall, these beams are required to support one-half panel of uniform floor load, $=15\times 2.5\times 105=3{,}940$ lbs., and the bending moment at the centre due to this load $=\frac{3{,}940\times 15}{8}=7{,}380$ ft.-lbs. Then the total centre moment $=31{,}140+7{,}380=38{,}520$ ft.-lbs. $=462{,}240$ in.-lbs.; and the S required $=462{,}240\div 16{,}000=28.89$. Thus a 12-in. I-beam at 31.5 lbs. per foot with $S=36$ is the proper section to use. A $12\times\frac{1}{2}$ in. plate will be rivetted to the top flange to carry the wall, and a $3\times 3\times\frac{1}{2}$ in. angle will be rivetted to the web on the inside to support the floor.

The connections of these beams to the column should be designed to carry one-half of the total weight of the wall (allowing for window openings) and the floor load. The wall load $=(14\times 7.5\times 120)-(8\times 3.5\times 120)=9{,}240$ lbs.; and the floor load $=7.5\times 2.5\times 105=1{,}960$ lbs. Then the total load on column connections $=9{,}240+1{,}960=11{,}200$ lbs.

Two beams, side by side and connected by cast iron separators, are frequently used for wall girders; but, when practicable, a single beam is more satisfactory, as it is more economical, and it requires less complicated connections to the columns.

End Wall Beams at First, Second, and Third Floors. Assuming the same arrangement of window openings as for the front and rear walls, the bending moment due to the weight of wall will also be the same; whereas the bending moment caused by the floor load will be equal to one-half of that already computed for the floorbeams of the upper floors. Then the total bending moment $=31{,}140+\frac{39{,}375}{2}=$ 50,825 ft.-lbs. $=609{,}900$ in.-lbs.; and the S required $=609{,}900\div 16{,}000=38.12$. This case calls for a 12-in. I-beam at 35 lbs. per ft. for which $S=38$. The wall load on column connections, as before, $=9{,}240$ lbs.; the floor load $=7.5\times 5\times 105=3{,}930$ lbs.; and the total load $=9{,}240+3{,}930=13{,}170$ lbs. A $12\times\frac{1}{4}$ in. plate will also be used on these beams to carry the wall.

Front and Rear Wall Beams at Ground Floor. The arrangement of the window openings may be taken as similar to that at the upper floors. The wall, however, is 18 ft. high; and the windows will be assumed 12 ft. high with 3 ft. of wall both above and below them. The load diagram will then be the same as in Fig. 58(*b*), except that the upper rectangle will be increased owing to the 4 ft. additional height of the piers between the windows. Thus the weight of the upper rectangle $=(4\times 12\times 120)+(3\times 7.5\times 120)=8{,}460$ lbs.; and the reaction at either end $=\frac{3{,}960+8{,}460}{2}+360=6{,}570$ lbs. The bending moment due to the wall loads is found as before, and that due to the floor load is equal to one-half the bending moment for ground floor stringers, as follows:

$$
\begin{array}{lrr}
6{,}570\times 7\frac{1}{2}= & & +49{,}275 \\
-\frac{8{,}460}{2}\times 1 = & -\ 4{,}230 & \\
-\frac{3{,}960}{2}\times 2\frac{3}{4} = & -\ 5{,}445 & \\
-\ 360\times 6\frac{1}{6} = & -\ 2{,}220 & -11{,}895 \\
\end{array}
$$

Moment at centre:

$$
\begin{array}{lcrr}
\text{Wall loads} & = & & +37{,}380 \\
\text{Floor load} & = & \frac{23{,}200}{2} = & 11{,}600 \\
\text{Total,} & & & 48{,}980 \text{ ft.-lbs.} = 587{,}760 \text{ in.-lbs.}
\end{array}
$$

and the S required $=587,760 \div 16,000=36.73$. A 12-in. I-beam at 35 lbs. per foot, for which $S=38.0$, will be used here, with a $12 \times \frac{1}{2}$ in. top plate. The wall load on column connections $=(18 \times 7.5 \times 120)-(12 \times 3.5 \times 120)=11,160$ lbs.; the floor load $=7.5 \times 2.5 \times 165=3,090$ lbs.; and the total load $=11,160+3,090=14,250$ lbs.

End Wall Beams at Ground Floor. The same arrangement of window openings as in front and rear walls is assumed, and thus the bending moment from wall load will be equal to 37,380 ft.-lbs., as above; while the bending moment due to floor load will be equal to one-half that obtained for the intermediate floorbeams, $=\frac{61,875}{2}=$ 30,940 ft.-lbs. The total centre moment $=37,380+30,940=68,320$ ft.-lbs. $=819,840$ in.-lbs.; and $819,840 \div 16,000=51.24$, the required S. A 15-in. I-beam at 42 lbs. per foot, and for which $S=58.9$, will be used, with a $12 \times \frac{1}{2}$ in. plate for wall as before. The wall load on column connections $=11,160$ lbs.; the floor load $=7.5 \times 5 \times 165=6,190$ lbs.; and the total load $=11,160+6,190=17,350$ lbs.

Loads on Columns. The interior columns support a roof and floor area equal to $15 \times 15=225$ sq.ft.; consequently, at the first, second, and third floors and roof, the loads imposed on these columns $=225 \times 105=23,625$ lbs.; and, at the ground floor $=225 \times 165=37,125$ lbs. The wall columns support a roof and floor area equal to $15 \times 7.5=112.5$ sq.ft.; thus, at the first, second, and third floors and roof, the loads $=112.5 \times 105=11,800$ lbs.; and, at the ground floor $=112.5 \times 165=18,560$ lbs. The wall loads carried by these columns are equal to the net area of one panel of wall multiplied by its weight per sq.ft. At the first, second, and third floors, the area of wall supported by each column $=(15 \times 14)-(2$ windows $3.5 \times 8)=154$ sq.ft.; and its weight $=154 \times 120=18,480$ lbs. At the ground floor, the area of wall supported by each column $=(15 \times 18)-(2$ windows $3.5 \times 12)=186$ sq.ft.; and its weight $=186 \times 120=22,320$ lbs.

SUMMARY OF LOADS ON INTERIOR COLUMNS

Roof load	23,625
Total load on columns between third floor and roof	23,625 lbs.
Third floor load	23,625
Total load on columns between second and third floors	47,250 lbs.

Second floor load	23,625
Total load on columns beween first and second floors	70,875 lbs.
First floor load	23,625
Total load on columns between ground and first floors	94,500 lbs.
Ground floor load	37,125
Total load on columns between basement and ground floors	131,625 lbs.

SUMMARY OF LOADS ON WALL COLUMNS

Roof load	11,800
Total load on columns between third floor and roof	11,800 lbs.
Third floor load	11,800
Wall load between third floor and roof	18,480
Total load on columns between second and third floors . . .	42,080 lbs.
Second floor load	11,800
Wall load between second and third floors	18,480
Total load on columns between first and second floors	72,360 lbs.
First floor load	11,800
Wall load between first and second floors	18,480
Total load on columns between ground and first floors . . .	102,640 lbs.
Ground floor load	18,560
Wall load between ground and first floors	22,320
Total load on columns between basement and ground floor	143,520 lbs.

The above total loads are all shown in the proper places in Fig. 57.

It is usually customary to make the individual sections of columns for this class of work long enough to extend through two stories, instead of splicing them at or near every floor. This arrangement requires some excess of metal over the amount that would be used if the section were altered at each floor to meet the exact requirements of the loads; but there are compensating advantages in shopwork and erection owing to the less number of pieces to handle. In the present example, the column splices are located 9 ins. above the first and third floor lines so as to clear the beam connections.

For the upper sections of the columns, Carnegie H-beams are used, the minimum size adopted being 6×6 ins. Referring to Table II, Art. 15, it will be found that the area of a 6×6 in. H-beam at 23.8 lbs. = 7.0 sq.ins., and that its least radius of gyration = 1.55 ins. Then, since the unsupported length of the column = 14 ft. = 168 ins., $\frac{l}{r}=\frac{168}{1.55}=108$, which ratio, by Table I, Art. 15, corresponds to a permissible unit-stress of 7,280 lbs. per sq.in.; and its capacity = 7,280×7.0 = 50,960 lbs. Although this section is capable of sustaining safely a much greater load than that on the columns above the third floor, a smaller shape is not recommended on account of the difficulty in making connections thereto.

Between the first and third floors, 8×8 in. H-beams at 34.6 lbs. are used. Their sectional area = 10.17 sq.ins., and their least radius of gyration = 2.01 ins. Then $\frac{l}{r}=\frac{168}{2.01}=83.5$, which, by Table I, corresponds to a permissible unit-stress of 8,650 lbs. per sq.in.; and the capacity of this section = 8,650×10.17 = 87,970 lbs., being considerably greater than the load on columns above the first floor.

Since there are no heavier sections of H-beams in the Table than the 8×8 in., a compound section of the form shown in Fig. 30 is used from the basement to the first floor, made up as follows:

4 angles $5\times3\frac{1}{2}\times\frac{5}{8}$ in.	= 12.20	($3\frac{1}{2}$-in. legs rivetted to web)
1 web-plate $8\times\frac{3}{8}$ in.	= 3.00	
	15.20 sq.ins.	

The least radius of gyration for this section, which is about the axis *cd*, is found by calculation to be 2.15 ins. Now the unsupported

length of columns between the ground and first floors = 18 ft. = 216 ins.; then $\frac{l}{r}=\frac{216}{2.15}=100$, which, by Table I, corresponds to a permissible unit-stress of 7,710 lbs. per sq.in.; and $7,710\times15.20=117,200$ lbs., the capacity of the columns. Between the basement and the ground floors, the unsupported length of the columns = 8 ft. = 96 ins.; and $\frac{l}{r}=\frac{96}{2.15}=45$, corresponding to a permissible unit-stress of 10,780 lbs. per sq.in.; and $10,780\times15.20=163,850$ lbs., which is the capacity of the columns between the basement and ground floors. Thus it will be seen that the section is ample for either the interior or wall columns.

As the loads on both the interior and the wall columns are approximately equal, their sections are made alike throughout.

Details. In Fig. 59 are shown some of the details for an interior and a wall column, as well as a floor stringer connection.

Beginning at the roof, it will be seen that the main roof beams rest directly on top of the columns and are connected thereto by two $3\frac{1}{2}\times3\frac{1}{2}\times\frac{3}{8}$ in. angles, 6 ins. long, the tops of the columns being faced to the bevel of the roof, and the connection angles bent correspondingly.

Splices. At the splices above the third floor, horizontal bearing plates $\frac{1}{2}$ in. thick are used to distribute the load from the upper sections over the lower, both upper and lower sections being faced to give perfect bearings. The $6\times\frac{3}{8}$ in. splice-plates on the flanges are intended to hold the two sections in line and to give stiffness to the joint. On account of the difference in width of the sections spliced, filler-plates are rivetted to the upper sections. The splices above the first floor should be designed similarly, except that the flange splices would be 8 ins. wide and no fillers are required.

Floorbeam Brackets. The maximum floorbeam reaction, which has been found in connection with the end wall beams at the ground floor = 17,350 lbs. Now, the value of a $\frac{3}{4}$-in. rivet in single shear (by Table V) = 5,300 lbs.; thus 4 rivets would be ample if they were subject only to direct shear. But there is a certain amount of bending moment on the bracket inducing tension on the heads of the upper rivets; and, on this account, the number of rivets should be somewhat increased. The drawing shows 8 rivets. In order to increase the general stiffness of the structure, the top flanges of the beams are connected to the columns by short $3\frac{1}{2}\times3\frac{1}{2}\times\frac{3}{8}$ in. angles as shown. All of the brackets, both for the 15-in. and the 12-in. beams, are made

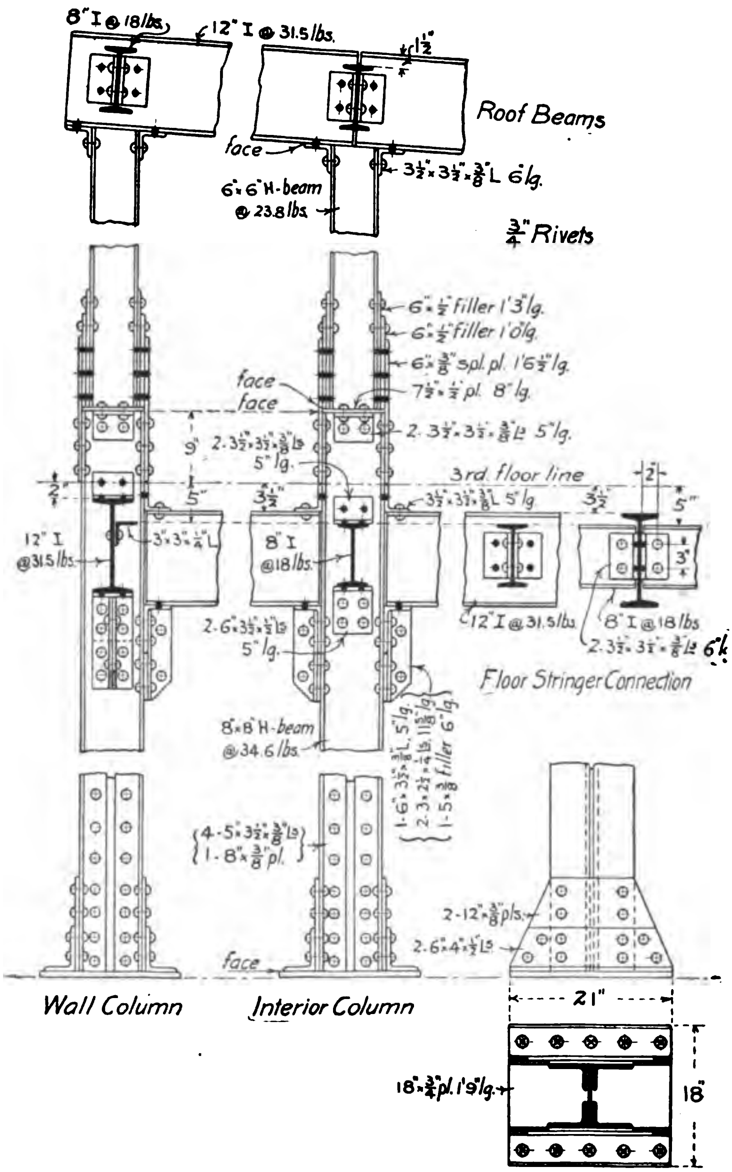

8" I @ 18 lbs.
12" I @ 31.5 lbs.
$1\frac{1}{2}$"
Roof Beams
face
6" x 6" H-beam @ 23.8 lbs.
$3\frac{1}{2}$" x $3\frac{1}{2}$" x $\frac{3}{8}$" L 6" lg.
$\frac{3}{4}$" Rivets
6" x $\frac{1}{2}$" filler 1' 3" lg.
6" x $\frac{1}{2}$" filler 1' 0" lg.
6" x $\frac{3}{8}$" spl. pl. 1' $6\frac{1}{2}$" lg.
$7\frac{1}{2}$" x $\frac{1}{2}$" pl. 8" lg.
face
face
2 - $3\frac{1}{2}$" x $3\frac{1}{2}$" x $\frac{3}{8}$" Ls 5" lg.
9"
2 - $3\frac{1}{2}$" x $3\frac{1}{2}$" x $\frac{3}{8}$" Ls 5" lg.
3rd floor line
2"
5"
$3\frac{1}{2}$"
$3\frac{1}{2}$" x $3\frac{1}{2}$" x $\frac{3}{8}$" L 5" lg.
12" I @ 31.5 lbs.
3" x 3" x $\frac{1}{4}$" L
8" I @ 18 lbs.
2 - 6" x $3\frac{1}{2}$" x $\frac{1}{2}$" Ls 5" lg.
12" I @ 31.5 lbs.
8" I @ 18 lbs.
2 - $3\frac{1}{2}$" x $3\frac{1}{2}$" x $\frac{3}{8}$" Ls 6" lg.
Floor Stringer Connection
8" x 8" H-beam @ 34.6 lbs.
1 - 6" x $3\frac{1}{2}$" x $\frac{3}{8}$" L, 5" lg.
2 - 3 x $2\frac{1}{2}$ x $\frac{1}{4}$ Ls, $11\frac{5}{8}$" lg.
1 - 5 x $\frac{3}{8}$ filler 6" lg.
4 - 5" x $3\frac{1}{2}$" x $\frac{3}{8}$" Ls
1 - 8" x $\frac{3}{8}$" pl.
2 - 12" x $\frac{3}{8}$" pls.
2 - 6" x 4" x $\frac{1}{2}$" Ls
face
Wall Column
Interior Column
21"
18" x $\frac{3}{4}$" pl. 1' 9" lg.
18"

Fig. 59.

alike to simplify the shopwork. For the 8-in. and 9-in. floor stringers, which are supported directly by the intermediate columns, lighter brackets are employed, consisting of $6 \times 3\frac{1}{2} \times \frac{1}{2}$ in. angles 5 ins. long, without vertical stiffeners.

Connections of Stringers to Beams at Upper Floors and Roof. The total load on a stringer, as computed near the beginning of this article, $=7,875$ lbs.; thus the end reaction $=7,875 \times \frac{1}{2}=3,940$ lbs. The stringer connections consist of two $3\frac{1}{2} \times 3\frac{1}{2} \times \frac{3}{8}$ in. angles, 6 ins. long, rivetted to the stringers by two $\frac{3}{4}$-in. rivets spaced 2 ins. from the back of the angles and 3 ins. apart vertically. Now there are two forces acting on these rivets: one vertical and the other horizontal. The vertical force is equal to the end reaction of the stringer and is distributed equally between the two rivets. The horizontal force is due to the fact that the end reaction is applied two inches, horizontally, from the rivets, thus causing a bending moment equal to the reaction multiplied by this distance $=3,940 \times 2=7,880$ in.-lbs.; and, since the resisting lever arm of the rivets $=3$ ins., the horizontal force on each $=7,880 \div 3=2,630$ lbs.; this force on the upper rivet acting away from the end of the beam, and that on the lower rivet acting towards it. The resultant force on each rivet is equal to the hypothenuse of a right-angled triangle, the horizontal side being equal to the horizontal force thus found, and the vertical side being equal to one-half of the end reaction. Then, the total stress on each rivet $=\sqrt{1,970^2+2,630^2}=3,280$ lbs. Since the webs of the 8-in. beams are $\frac{1}{4}$ in. thick, the minimum value of a $\frac{3}{4}$-in. rivet, by Table V, $=4,500$ lbs., which is greater than the stress, and therefore the connection is amply strong. The four field rivets which pass through the webs of the floorbeams are required to support the ends of the two adjacent stringers, and thus the load on them $=7,875$ lbs. The floorbeam webs are $\frac{3}{8}$ in. thick, and thus the bearing value of each $\frac{3}{4}$-in. rivet, by Table VI, $=5,070$ lbs., which is much greater than the stress attributed to it.

Bases of Columns. The maximum column load, as shown in Fig. 57, $=143,520$ lbs.; and, with a permissible bearing of 400 lbs. per sq.in. on the concrete foundations, the bearing area required $=143,520 \div 400=359$ sq.ins. The area of the base plates used $=18 \times 21=378$ sq.ins. Two side plates $12 \times \frac{3}{8}$ ins. and two angles $6 \times 4 \times \frac{1}{2}$ ins. are used to assist in distributing the loads from the columns over the base plates.

Wind Bracing. In small buildings, similar to the example here dealt with, the curtain walls between the outer columns provide ample

wind bracing; but in designing sky-scrapers special bracing must be supplied to resist the wind forces.

This article is only given as an introduction to office building construction; but it should serve to indicate to the student the general principles involved. In large buildings many complicated problems arise, which must be studied independently, and can only be dealt with when all of the conditions are known.

CHAPTER VII

THE DESIGN OF A SIMPLE ROOF TRUSS

THE roof truss considered in this chapter is required for a brick building 40 ft. wide c. to c. of walls. The walls are assumed to be 13 ins. thick, with pilasters 21×21 ins. at the trusses, which latter are 15 ft. apart c. to c. The trusses are of the Fink pattern, with the rafters divided into 8 equal panels, the angle between the rafters and the horizontal being 30°. The roof is covered with slates laid on 2-in. boards, and the latter are supported by longitudinal steel purlins placed at the panel-points. In accordance with the data and rules given in Art. 17, the weight of the trusses per sq.ft. of horizontal area is assumed at 5 lbs., and the steel purlins at 5 lbs. The weight of the roof boards per sq.ft. of roof area is taken at 5 lbs.; the slates at 10 lbs., and the snow-load at 15 lbs. It will be sufficiently accurate to add the weight of the trusses and purlins per sq.ft. of horizontal area to the weight of the boards, slates, and snow for the total weight per sq.ft. of roof area. Thus the total assumed vertical load = 40 lbs. per sq.ft. of roof area. The wind-load, which is normal to the plane of the roof, and acts only on one side at a time, = 20 lbs. per sq.ft.

Panel Loads. Both the vertical and the wind-loads are assumed to be concentrated at the panel-points of the rafters, which are about 5.75 ft. apart; thus the intermediate panel-points support a roof area of $5.75 \times 15 = 86.25$ sq.ft.; and the end panel-points one-half of this amount. Then, for the vertical load, the intermediate panel-point concentrations $= 86.25 \times 40 = 3{,}450$ lbs.; and the end panel-point concentrations $= 3{,}450 \times \frac{1}{2} = 1{,}725$ lbs. For the wind-load, the intermediate concentrations $= 86.25 \times 20 = 1{,}725$ lbs.; and the end concentrations $= 1{,}725 \times \frac{1}{2} = 860$ lbs. The loads are shown on the truss diagram in Fig. 60.

Stress-Diagrams. The vertical and the wind-loads are treated separately. The stress-diagram for the former requires no explanation

in this place, as it is fully described in Art. 3. The resultant of the wind-loads is applied at the middle point of the rafter; and, when produced, cuts the horizontal member in a point distant one-third of

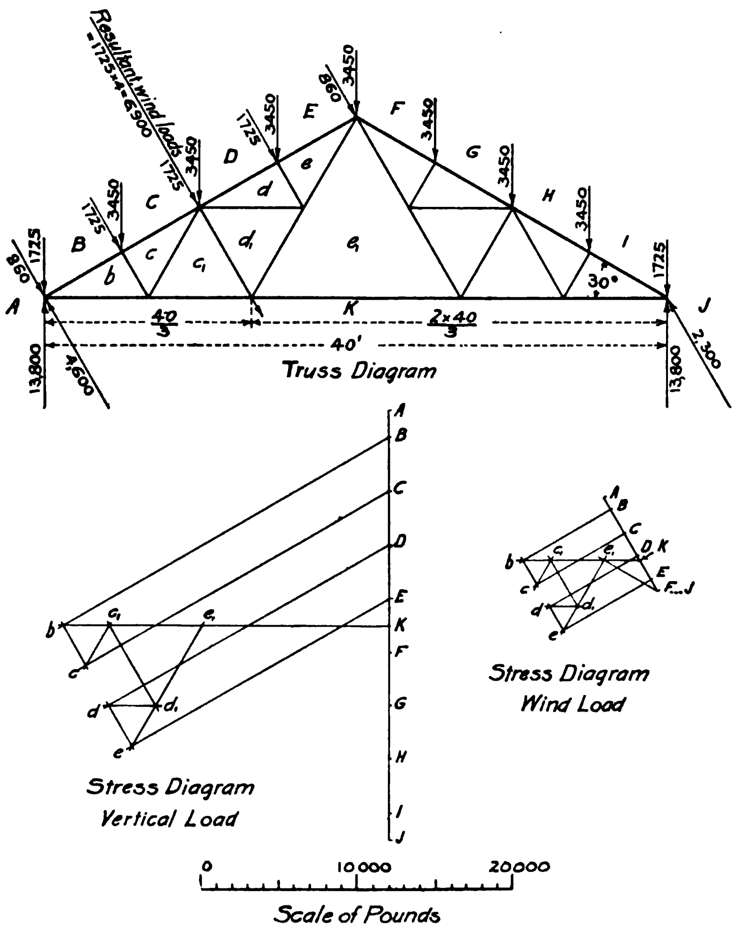


FIG. 60.

the span from the left-hand end, as shown. Therefore, the reactions at the right-hand and left-hand supports are equal respectively to one-third and two-thirds of the total wind-load, viz., 2,300 lbs. and 4600 lbs.; and their direction is parallel but opposite to the applied force. The wind-loads AB, BC, CD, DE, EF, . . . J are laid off

on the load line in regular order downwards; and the reactions *JK* and *KA* upwards. The stress-diagram is then proceeded with as in previous examples. It should be noted that there are no stresses in the web-members of the leeward half of the truss, but only in the rafter and bottom chord. The vertical load stresses as well as the maximum wind-load stresses are now scaled from the diagrams, and summarized in Fig. 61; the former being indicated by the letter *v*, and the latter by *w*.

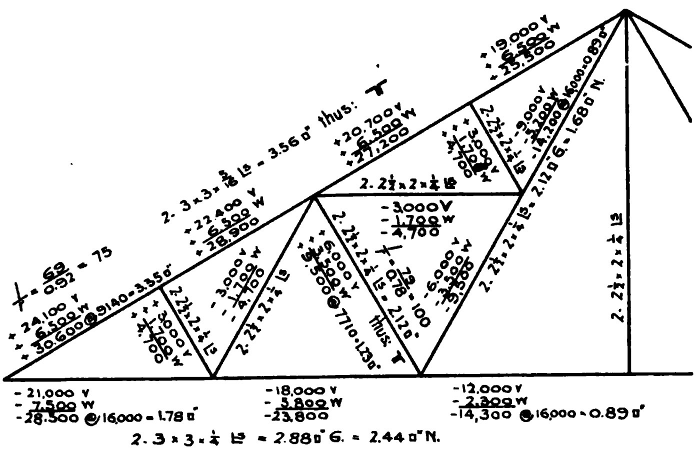


FIG. 61.

Proportioning of Truss Members. The required sectional areas for the tension members are obtained directly by dividing the various total stresses by the permissible unit-stress of 16,000 lbs. per sq.in., as shown. Suitable angles are then selected, making due allowance for rivet holes. In members subject to small stress requiring rivets in one leg of the angles only, the area of one hole should be deducted from each angle; but, when angles are connected by both legs, it is advisable to allow for two holes in each angle. Angles requiring more than three or four rivets should, when practicable, be connected by both legs. In the example, $\frac{3}{4}$-in. rivets are used; therefore, as explained in Art. 19, allowance should be made for $\frac{7}{8}$-in. holes. Tables of two angles, giving their gross and net areas, allowing for one and two holes in each angle, will be found of great assistance in this work. All members of the truss are composed of two angles each, and the minimum size used is $2\frac{1}{2}\times 2\times\frac{1}{4}$ ins.

In the end panel of the bottom chord, the stress is $-28,500$. Then $28,500 \div 16,000 = 1.78$ sq.ins. required; and the section provided is as follows:

Two angles $3 \times 3 \times \frac{1}{4}$ in.	$= 2.88$
Less two holes $\frac{7}{8} \times \frac{1}{4}$ in.	$= 0.44$
Net area	$= 2.44$ sq.ins.

This section is used throughout the bottom chord, for the sake of uniformity in appearance and to avoid changes in the rivet gauge lines, even though much smaller angles would be sufficient to withstand the stress in the centre panel.

In the long diagonal web-members which extend from the bottom chord to the apex of the rafters, the maximum stress is $-14,200$. Then $14,200 \div 16,000 = 0.89$ sq.ins. required; and the section provided is as follows:

Two angles $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{1}{4}$ in.	$= 2.12$
Less two holes $\frac{7}{8} \times \frac{1}{4}$ in.	$= 0.44$
Net area	$= 1.68$ sq.ins.

In the principal compression members of the web system, which extend from the middle point of the rafters to the bottom chord, the maximum stress is $+9,500$, and their length is about 79 ins. Assuming 2 angles $2\frac{1}{2} \times 2 \times \frac{1}{4}$ ins., with the longer legs back to back but separated sufficiently to straddle the connection plates, as shown, the least radius of gyration, which may be computed or taken from tables of radii of gyration found in the handbooks of the rolling mills, is found to be 0.78 in. Then the ratio $\frac{l}{r} = \frac{79}{0.78} = 100$, which, by Table I, corresponds to a permissible unit-stress of 7,710 lbs. per sq.in.; and $9,500 \div 7,710 = 1.23$ sq.ins. required. The sectional area of the two angles assumed, which is 2.12 sq.ins., is therefore ample.

For the other web-members, which are subject to smaller stresses, two angles $2\frac{1}{2} \times 2 \times \frac{1}{4}$ ins., are also used. The duty of the vertical member at the centre of the truss is merely to prevent excessive sag in the middle panel of the bottom chord.

In the end panels of the rafters, the maximum stress is $+30,600$, and the length c. to c. of panel-points is 69 ins. Assuming 2 angles

$3 \times 3 \times \frac{5}{16}$ ins., the least radius of gyration is found to be 0.92 in. Then $\frac{l}{r}=\frac{69}{0.92}=75$, which corresponds to a permissible unit stress of 9,140 lbs. per sq.in.; and $30,600 \div 9,140 = 3.35$ sq.ins. required. The area of the angles assumed, which is 3.56 sq.ins., is thus sufficient. The same section is used throughout the rafters.

Purlins. The purlins, which are set normal to the plane of the rafters, will be designed for a total load of 50 lbs. per sq.ft. of roof area supported. The span of the purlins = 15 ft., and their distance apart = 5.75 ft.; thus the load on each purlin $= 15 \times 5.75 \times 50 = 4310$ lbs. The bending moment $=\frac{4,310 \times 15}{8}=8,080$ ft.-lbs. $=96,960$ in.-lbs.; and the section modulus required $= 96,960 \div 16,000 = 6.06$. Turning to the tables of properties of standard channels given in the handbooks of the rolling mills, it will be found that a 7-in. channel, weighing 12.25 lbs. per foot and having a section modulus of 6.9, is suitable. The reasons for using channels instead of I-beams in this case are because of the greater ease with which the former can be attached efficiently to the rafters, and also in order to permit of wooden nailing strips on their back for the roof planking. Owing to the inclination of the channels from the vertical, they are subject to lateral bending, which, however, is amply provided for by the solid planking of the roof.

Details. In Fig. 62 are shown the details for one-half of a truss. The truss is provided with field joints, as shown, for convenience in shipment. The gusset-plates are all $\frac{7}{16}$ in. thick; and the rivets used are $\frac{3}{4}$ in. in diameter, which is the largest size permissible in $2\frac{1}{2}$ in. angles. The values of the shop-driven rivets are obtained in Table V; and of the field-driven rivets, in Table VI, Chapter V, Art. 19. Then in single shear a shop-driven rivet is good for 5,300 lbs., and a field-driven rivet, for 3,975 lbs.; in bearing on $\frac{7}{16}$-in. plate, a shop-driven rivet is worth 7,980 lbs., and a field-driven rivet, 5,910 lbs.

At the ends of the truss, the stress in the bottom chord $= -28,500$ lbs., which, at 7,980 lbs. per rivet, = 4 rivets required. The stress in the rafters $= +30,600$ lbs., also requiring 4 rivets. Now, referring to Fig. 60, it will be seen that the reaction for the vertical loads = 13,800 lbs.; and for the wind loads 4,600 lbs. The vertical component of the latter is obtained by multiplying by the cosine of 30°, thus: $4,600 \times 0.866 = 3,980$ lbs. Then the total vertical reaction $= 13,800 + 3,980 = 17,780$ lbs.; and, provided that the bottom edge of the gusset-plate does not bear perfectly on the shoe-plate, this force must be transmitted

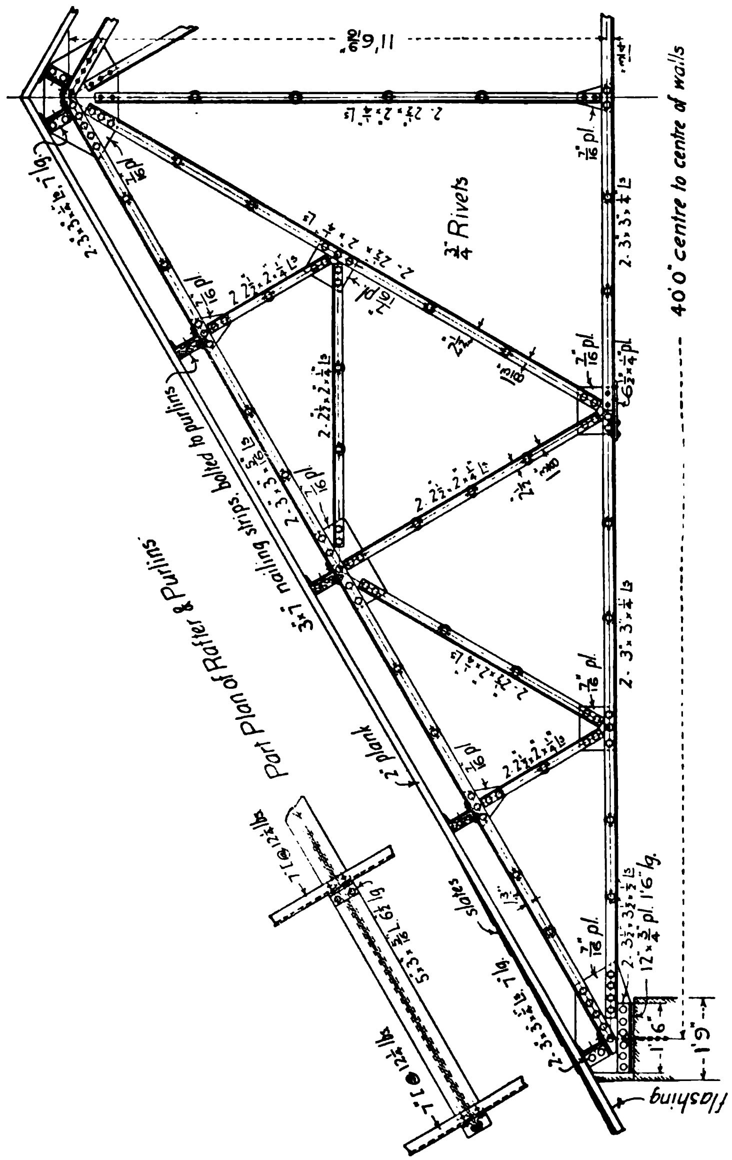

Part Plan of Rafter & Purlins
3/4" Rivets
40'0" centre to centre of walls
2" plank
slates
flashing

FIG. 62.

through the $3\frac{1}{2}\times3\frac{1}{2}\times\frac{1}{2}$ ins. connection angles and the rivets therein. Consequently, the number of rivets required in the vertical legs of these angles $=17,780\div7,980=3$. In the detail drawing, this number is largely exceeded; but, according to what is generally accepted as good practice, the rivets here should not be further apart than about 3 ins. Now, assuming that the walls are of common brick laid in lime mortar, the bearing pressure on them should not exceed 100 lbs. per sq.in.; thus the area required in shoe-plates $=17,780\div100=178$ sq.ins. The area of the plates used $=12\times18=216$ sq.ins. The bearing plates and their connection angles are made thick enough to distribute the load over the brickwork satisfactorily. The object of extending the gusset-plates above the rafters and below the bottom chord is to avoid concentrating the stresses from these members at the upper and lower edges of the said gusset-plates.

At the apex of the truss, the members on one side of the centre line are shop-rivetted, while those on the other side are field-rivetted; therefore, since it is desirable to make the joint symmetrical, the value of a field-driven rivet should be used in determining the number required. The maximum stress in the rafters $=+25,500$, which, at 5,910 lbs. per rivet $=5$ rivets required. The maximum stress in the web-members connected at this point $=-14,200$, requiring 3 rivets. As at the ends of the truss, the gusset-plate is extended above the rafters to avoid the concentration of stress in its upper edges.

At the joint adjacent to the centre panel of the bottom chord, the stresses in members c_1d_1 and d_1e_1 are each 9,500 lbs., requiring 2 rivets. In the bottom-chord member Kc_1, the stress $=23,800$ lbs., and the value of the connection is as follows:

Two shop rivets in bearing on $\frac{7}{16}$-in. plate at $7,980=15,960$
Four field rivets in bearing on $\frac{1}{4}$-in. plate at $3,375=13,500$

29,460 lbs.

In the bottom-chord member e_1K, the stress $=14,300$ lbs., and the value of the connection is:

Two field rivets in bearing on $\frac{7}{16}$-in. plate at $5,910=11,820$
Four field rivets in bearing on $\frac{1}{4}$-in. plate at $3,375=13,500$

25,320 lbs.

It should be noted that the bearing value of a $\frac{3}{4}$-in. rivet on $\frac{1}{4}$-in. plate is less than its single shearing value.

In the short members bc, cc_1, dd_1, de, the total stress is 4,700 lbs., requiring only one rivet; but at least two rivets should be used in every connection, no matter how small the stress.

The thickness of the gusset-plates, at the ends of the truss and at the apex, has been determined in this example by the permissible bearing value of the rivets therein, and the total stresses in the members connected thereby, with the object of making these plates as small as practicable. The intermediate gusset-plates are made of the same thickness as the others in order to keep the rafters and bottom-chord of uniform width.

Each purlin is connected to the rafters by two rivets: one through the web of the channel, and one through the bottom flange. The 3×7-in. nailing strips are intended to be connected to the channels by $\frac{1}{2}$-in. bolts about 18 ins. apart.

Lateral Bracing in the planes of the rafters should be provided in at least two panels of the building; and may consist of light angles or $\frac{3}{4}$-in. round rods, connected by gusset-plates attached to the horizontal legs of the rafters at or near the end purlins, the purlins at the middle-point of the rafters and at the apex. This lateral bracing is required principally during erection, and is of little or no use after the roof planking is laid.

CHAPTER VIII

The Design of a Roof Truss Supported by Steel Columns

The roof truss and supporting columns here considered are intended for an ordinary mill building, the principal data being as follows:

Width, c. to c. of columns, 60 ft.
Height from floor line to centre of bottom chord, 20 ft.
Roof trusses, divided into 12 panels of 5 ft.; depth at ends 4 ft.; at first panel-point from ends, 5 ft.; at centre, 6 ft. 6 ins.
Distance between trusses, 15 ft.
Curtain walls between columns, of brick 12 ins. thick.
Roof of solid timber construction 5 ins. thick, covered with tar and gravel.

Loads for Roof Trusses.

Dead-load:	Trusses	5
	Roof planking..	15
	Total	20 lbs. per sq.ft. of roof area.
Live-loads:	Snow	30 lbs. per sq ft. of roof area.
	Trolley	10,000 lbs., concentrated at any point of bottom chord.
Wind-load		30 lbs. per sq.ft., acting on side of building.

Panel-Loads. Both the dead- and snow-loads are assumed to be concentrated at the upper panel-points which are 5 ft. apart; thus the intermediate panel-points support an area of $15\times5=75$ sq.ft.; and the end panel-points, one-half of this amount. Then, for the dead-

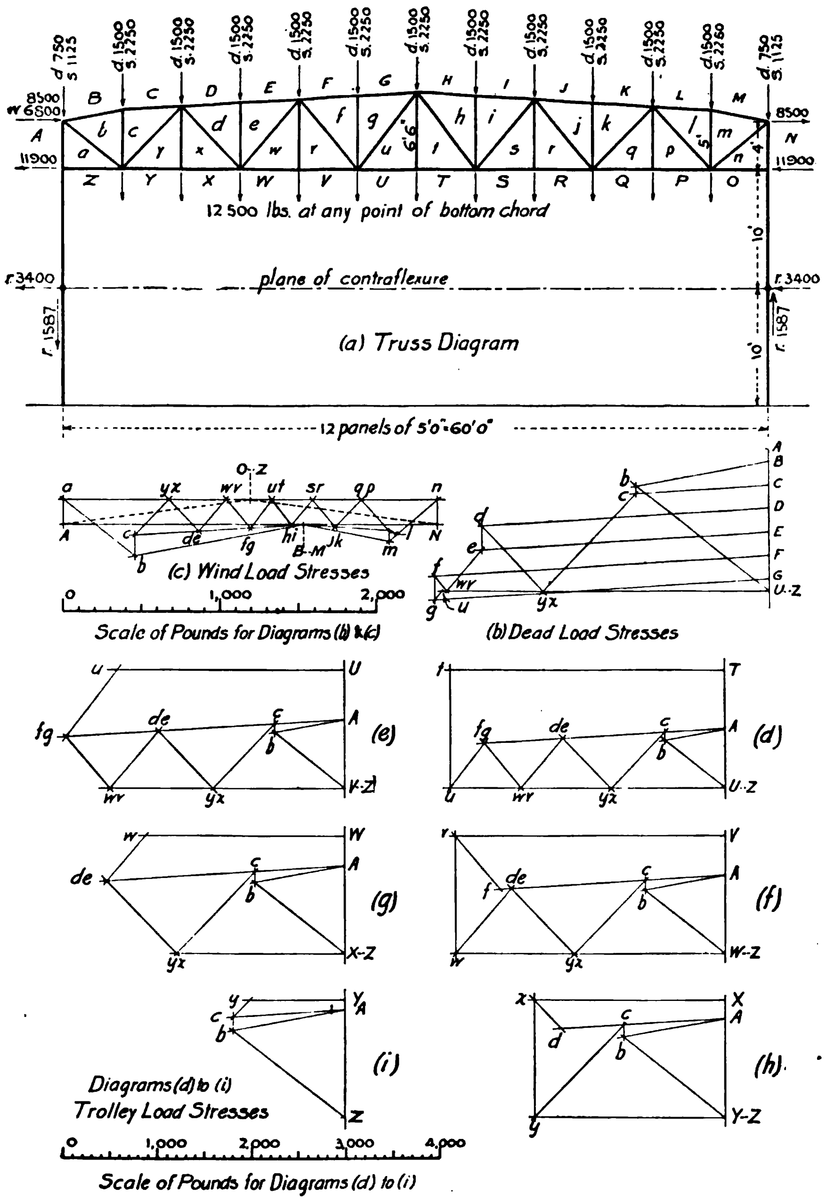

12 500 lbs. at any point of bottom chord
plane of contraflexure
(a) Truss Diagram
12 panels of 5'0"=60'0"
(c) Wind Load Stresses
Scale of Pounds for Diagrams (b) & (c)
(b) Dead Load Stresses
Diagrams (d) to (i)
Trolley Load Stresses
Scale of Pounds for Diagrams (d) to (i)

FIG. 63.

load, the intermediate panel-point concentrations $=75\times20=1,500$ lbs.; and the end panel-point concentrations $=1,500\times\frac{1}{2}=750$ lbs. For the snow-load, the intermediate panel-point concentrations $=75\times30=2,250$ lbs.; and the end panel-point concentrations $=2,250\times\frac{1}{2}=1,125$ lbs. These dead- and snow-load concentrations are shown in Fig. 63(a), the former being indicated by the letter d, and the latter, by the letter s.

Wind Force. The wind force is assumed to act on the exposed wall area, as well as on the vertical projection of the sloping roof. Now, since the total height of posts $=24$ ft., and the distance between them, longitudinally, $=15$ ft., the wind-load on one post $=24\times15\times30=10,800$ lbs.; one-half of which, $=5,400$ lbs., may be considered as concentrated at the top of the post. The other half of this load, which may be considered as concentrated at the base of the post, is neglected because it has no tendency to overturn the building. The height from the top of the posts to the top of the roof is about 3 ft.; thus the horizontal wind-load on one truss $=3\times15\times30=1,350$ lbs. This will also be assumed to be concentrated at the top of the post; and the total horizontal wind force at this point becomes $5,400+1,350=6,750$ lbs., which, to give round numbers, is taken at 6,800 lbs., as shown, being indicated by the letter w.

Trolley-Load. The trolley-load of 10,000 lbs., to which is added 25 per cent in order to provide for the effect of impact and vibrations produced thereby, thus increasing it to 12,500 lbs., is assumed to be concentrated at any point of the bottom chord.

Dead-Load Stresses. Since the truss and its loads are symmetrical it is only necessary to construct the stress-diagram for one-half of the frame; thus, in Fig. 63(b), the loads A to G, and one-half of the centre load GH, are laid off on the vertical load line to a convenient scale of pounds, as shown, the reaction at the left-hand end being $U \ldots ZA$. The stress-diagram is then proceeded with, beginning at the point where the post aA meets the members Bb and ba, the stresses in these latter members being the only unknown forces. For this condition of loading there are no stresses in the vertical members yx, wv, ut, sr, qp, nor in the end panels of the bottom chord nO and Za.

Snow-Load Stresses. For the snow-load, the conditions are exactly the same as for the dead-load. It is therefore unnecessary to construct a separate stress-diagram for this case, as the stresses may readily be determined by proportion from the dead-load stresses; or, since the snow-load concentrations are exactly 50 per cent greater than those of the dead-load, the stresses will also be 50 per cent greater.

Wind-Load Stresses. It is assumed that the posts are efficiently fixed at their base by anchor bolts or otherwise, so that the bending moment at this point, due to the application of a horizontal force to the truss, will be approximately equal but opposite to that at the point of connection to the bottom chord. Thus there will be a point of no moment, or of contraflexure, about midway between the lower extremity of each post and the point where it is fixed by the bottom chord. In Fig. 63(*a*) the points of contraflexure are indicated by small circles, located 10 ft. from the base of the columns, and the same distance from the bottom chord of the truss; and the plane passing through these points is called the plane of contraflexure.

If the posts were hinged at their base, they would be incapable of resisting bending moments about these points; and the vertical reactions due to the horizontal wind force would be obtained by taking moments of this force about the lower (or free) ends of the posts, and dividing by their distance c. to c. But, since the posts are to be fixed at their base, they can rotate only at their points of contraflexure, which may be considered as hinged joints; and, therefore, to obtain the vertical reactions due to a horizontal force, moments of this force should be taken about the points of contraflexure (or hinges), and divided by the distance c. to c. of posts, as before. Now, the vertical height of the applied wind force of 6,800 lbs. above the plane of contraflexure $=14$ ft., and the distance between the posts$=60$ ft.; then the vertical reactions$=\frac{6,800\times 14}{60}=1,587$ lbs., the reaction at the leeward post being positive, and that at the windward post negative.

It is further assumed that the horizontal wind force of 6,800 lbs., which is applied at the top of the left-hand post, is resisted equally by both posts; thus the horizontal reactions at their base are each equal to $6,800\times\frac{1}{2}=3,400$ lbs. Although this is the usual assumption, its correctness has been questioned by some authorities, who maintain that the horizontal reaction at the base of the leeward post is less than that at the base of the windward post, owing to the distortions of the truss members through which a portion of the applied load must pass. But the author believes that the assumption of equal horizontal reactions is as nearly correct as any other; for it has been determined experimentally that the suction on the leeward side of a building is as great, if not greater than the applied force on the windward side. It might therefore be more consistent to apply one-half of the wind force of 6,800 lbs. to the top of each post instead of applying the whole of it

to the top of the windward post; but this would make little difference in the resulting stresses, and the general method adopted undoubtedly gives satisfactory and safe results.

The reactions, at the points of contraflexure, for the wind force are indicated on the diagram, Fig. 63(*a*), by the letter *r*. The horizontal reactions at both posts act in the opposite direction to the applied force. The vertical reaction at the right-hand post acts upwards, and that at the left-hand post, downwards.

Now, as before stated, the bending moments at the base of the posts and at the point where they are fixed by the bottom chord of the truss are of the same magnitude but opposite in direction, being each equal to the horizontal shear or reaction at the points of contraflexure multiplied by its common distance from either of the fixed points mentioned. Thus the bending moment at the base of posts and at the bottom-chord connection $=3{,}400\times 10=34{,}000$ ft.-lbs. The moment at the base is resisted principally by the anchor bolts, and will be considered later; while that at the bottom-chord connection is resisted by an induced horizontal force acting at the top of the post with a lever-arm of 4 ft. Therefore, the force required at the top of the post to resist the moment at the bottom-chord connection $=\frac{34{,}000}{4}=8{,}500$ lbs. This force acts in the same direction as the horizontal reaction at the point of contraflexure, and must be balanced by an equal but opposite force acting in the direction of the applied wind force, as shown. At the bottom-chord connection there is a force equal to the sum of the two forces at the ends of the lever, viz., the reaction at the point of contraflexure and the induced force at the top $=3{,}400+8{,}500=11{,}900$ lbs. This force acts in the opposite direction to the two forces which it balances, and must be resisted by an equal force at this point, acting towards the left, as shown.

Having supplied the forces at the top of both posts and at the point of their connection with the bottom chord of the truss, which are required for equilibrium, the stress-diagram may now be proceeded with, as follows: In Fig. 63(*c*), the external forces are taken in regular order in going around the roof truss in a clockwise direction, beginning with the force AB, which consists of the wind load of 6,800 lbs., together with the force of 8,500 lbs. supplied to balance the internal stress induced by the bending action of the post. The next force $MN=8{,}500$ lbs., which is applied at the top of the leeward post, acts in the same direction as AB. The force NO is made up of two components: the horizontal

force of 11,900 lbs., which acts towards the left; and the vertical reaction of 1,587 lbs., which acts upwards. Finally, the diagram of external forces is closed by the force *ZA*, also made up of two components: the vertical reaction of 1,587 lbs., action downwards; and the horizontal force of 11,900 lbs., acting towards the left.

Now, at the point where the left-hand post is connected to the bottom-chord of the truss, there are but two unknown forces: *Aa* and *aZ*; and these are obtained by drawing, from the points *A* and *Z* in the stress-diagram, lines parallel to the members *Aa* and *aZ*, which lines intersect in the point *a*. In going around this joint in the same direction as that in which the external forces have been laid off, and following the direction of the forces in the stress-diagram, it will be observed that *Aa* and *aZ* both act away from the joint, thus indicating tension in the corresponding members.

The joint at the top of the left-hand post should be considered next, where there are now two known forces: *aA* and *AB*, and but two unknown forces: the stresses in members *Bb* and *ba*, which are obtained by drawing, from the points *B* . . . *M* and *a* in the stress-diagram, lines parallel to the members *Bb* and *ba*. The lengths of these two lines are determined by their intersection in the point *b*. Now, taking the members at this joint in regular order, as before, and following the direction of the corresponding forces in the stress-diagram, *Bb* and *ba* are both found to act towards the point considered, which indicates that these forces represent compression.

At the upper end of the vertical member *bc* there are now but two unknown forces. Then, from the point *B* . . . *M* in the stress-diagram, a line is drawn parallel to the member *Cc*; and, from the point *b* a line parallel to the member *cb*, which two lines intersect in the point *c*. Both *Cc* and *cb* represent compression.

It will be unnecessary to give in detail the operation to be performed at each succeeding panel-point, as the work is all similar. The diagram must be constructed for the whole truss, because the stresses in the members at opposite ends are entirely different—both in magnitude and kind. Since for this condition of loading there are no external forces between *B* and *M*, these and the intermediate letters indicate a single point in the stress-diagram; and the same explanation applies to the letters *O* to *Z*. There being no wind-load stresses in the vertical members *de*, *fg*, *hi*, *jk*, *pq*, *rs*, *tu*, *vw*, and *xy*, these various pairs of letters also indicate but single points in the stress-diagram, as shown. If the diagram does not close exactly at first, it may be necessary to

go over it again, or to work backwards from the right-hand end of the truss until the figure closes.

Trolley-Load Stresses. In order to obtain the maximum stresses in the various members of the truss due to the trolley-load, it is necessary to make a separate stress-diagram for this load when placed at each panel-point between the centre of the span and either end. Before constructing the stress-diagrams, however, the reactions for the load in its various positions must be calculated, thus:

For load TU, reaction $ZA = 12{,}500 \times \frac{6}{12} = 6{,}250$ lbs.
" UV, reaction $ZA = 12{,}500 \times \frac{7}{12} = 7{,}290$ "
" VW, reaction $ZA = 12{,}500 \times \frac{8}{12} = 8{,}330$ "
" WX, reaction $ZA = 12{,}500 \times \frac{9}{12} = 9{,}375$ "
" XY, reaction $ZA = 12{,}500 \times \frac{10}{12} = 10{,}420$ "
" YZ, reaction $ZA = 12{,}500 \times \frac{11}{12} = 11{,}460$ "

Diagram (*d*) Fig. 63, is for the load at the centre, viz., TU. This load is laid off downwards, and the reaction ZA, upwards. It should be noted that, since for these cases there are no loads applied to the upper panel-points, the capital letters B to N on the truss diagram have no significance. At the upper end of the left-hand post there are but two unknown forces: the stresses in Ab and ba (or bZ, since there is no stress in aZ). The two lines intersect in the point b, and so determine the stresses required. This diagram must be carried far enough to give the stresses in the centre members of the truss; and from it the maximum compression in gu is obtained, as well as the maximum tension in uU.

Diagram (*e*), which is for the load UV, gives the maximum compression in Ff and Gg; also the maximum tension in vf and gu.

Diagram (*f*), which is for the load VW, gives the maximum compression in members ew and vf; also the maximum tension in vV and Ww.

Diagram (*g*), which is for the load WX, gives the maximum compression in Dd and Ee; also the maximum tension in xd and ew.

Diagram (*h*), which is for the load XY, gives the maximum compression in Bb and Cc; also the maximum tension in ab and cy.

Diagram (*i*), which is for the load YZ, gives the maximum compression in Bb and Cc; also the maximum tension in ab and cy.

In order to keep the diagrams (*d*) to (*i*) within convenient dimensions, they are constructed to a smaller scale than diagrams (*b*) and (*c*).

Combination of Stresses. Owing to the large number of stresses to be summarized, it would be found inconvenient to do so on a small diagram of the truss, for lack of available space; consequently, they will be arranged in the form of a table. Since the wind-load stresses in corresponding members at opposite ends of the truss are different in kind as well as in magnitude; and since, with the wind in the opposite direction to that assumed, the conditions will be reversed; therefore, the stresses obtained from the diagram for the right-hand end of the truss are applied to the corresponding members of the left-hand end, as well as the stresses attributed to these members, the object being to obtain the maximum compression and tension which can occur in every member with any possible combination of the assumed loads.

TABLE OF STRESSES IN POUNDS

Members.	Dead-Load.		Snow-Load.		Wind-Load.		Trolley-Load.		Totals.	
	+	−	+	−	+	−	+	−	+	−
Aa	9,000	....	13,500	....	1,600	1,600	12,500	....	36,600	
Bb	8,600	....	12,900	....	10,900	5,600	12,000	....	44,400	
Cc	8,500	....	12,750	....	10,700	5,500	11,900	....	43,850	
Dd and *Ee*	18,300	....	27,450	....	6,700	2,000	25,300	....	77,750	
Ff and *Gg*	21,300	....	31,950	....	3,400	....	29,600	....	86,250	
Tt and *Uu*	....	20,800	....	31,200	1,400	....	....	29,200	....	81,200
Vv and *Ww*	....	20,500	....	30,750	4,000	1,600	....	28,500	....	81,350
Xx and *Yy*	....	14,400	....	21,600	7,100	5,200	....	20,200	....	61,400
Za	....	....	....	....	11,900	11,900	....	....	11,900	11,900
ab	....	10,800	....	16,200	5,800	4,000	....	15,000	....	46,000
bc	400	....	600	....	700	1,300	....	1,400	1,100	2,300
cy	8,600	....	12,900	....	2,600	3,100	13,600	2,500	37,700	
xd	....	5,700	....	8,550	2,800	2,400	4,100	10,700	1,200	27,350
ew	3,400	....	5,100	....	2,200	2,600	9,100	6,500	19,800	5,700
vf	....	1,200	....	1,800	2,400	2,100	7,500	7,200	8,700	12,300
gu	....	800	....	1,200	2,000	2,200	6,000	8,900	7,200	13,100
de and *fg*	1,500	....	2,250	....	....	....	....	....	3,750	
tu, *vw*, and *xy*	....	....	....	....	....	....	....	12,500	....	12,500

As the dead-load acts at all times, it must be included in every combination. In the table the plus sign represents compression, and the minus sign tension.

Bending Moments in Top Chord. Owing to the fact that the roof covering is supported directly by the top chord of the truss, instead of being applied to the panel-points by means of purlins, it induces bending moments in this member in addition to the direct or axial stresses which have already been computed; and these bending moments

must be taken into account in the determination of the section required. Now the length of one panel of the truss = 5 ft.; the distance c. to c. of trusses = 15 ft.; and the total roof load, including snow but omitting the weight of the truss, = 45 lbs. per sq.ft.; thus the load on one panel $= 5 \times 15 \times 45 = 3{,}375$ lbs. The conditions are similar to those for a beam fixed at both ends and uniformly loaded (Art. 14, Case VII); and the maximum moment, which is at the ends, is equal to $\frac{2}{3}$ of that which would be obtained for a simple span of the same length. Therefore, the bending moment in any panel of the top chord

$$= \frac{3{,}375 \times 5}{8} \times \frac{2}{3} = 1{,}400 \text{ ft.-lbs.} = 16{,}800 \text{ in.-lbs.}$$

Bending Moments in Bottom Chord. For a concentrated load P at the middle point of a beam of length l simply supported at the ends, the bending moment (Art. 6) $= \frac{Pl}{4}$; and, if the same beam were rigidly fixed at the ends, the bending moment would be equal to $\frac{Pl}{8}$. (Art. 14, Case VI). The individual members of the bottom chord, however, are not free at the panel-points, neither are they absolutely fixed; for, unlike the top chord, the remaining panels are unloaded. Consequently, the bending moment at the middle point must be somewhere between these two extremes; and it may be proven that it is approximately equal to $\frac{Pl}{6}$. Therefore, the bending moment in any panel of the bottom chord

$$= \frac{12{,}500 \times 5}{6} = 11{,}400 \text{ ft.-lbs.} = 136{,}800 \text{ in.-lbs.}$$

Proportioning of Parts. The total stresses, which have already been computed, as well as the sections provided are shown in Fig. 64. The sectional areas for the various members are determined as follows:

Web Members. In member *ab*, the stress is $-46{,}000$; and $46{,}000 \div 16{,}000 = 2.88$ sq.ins. required. The section used is composed of

Two angles $4 \times 3 \times \frac{5}{16}$ in.	= 4.18
Less four holes $\frac{7}{8} \times \frac{5}{16}$ in.	= 1.10
Net area.	= 3.08 sq.ins.

These angles will be connected by both legs, and so allowance is made for two rivet holes in each angle, assuming $\frac{3}{4}$ in. rivets and $\frac{7}{8}$ in. holes, in accordance with Art. 19.

In member *bc* the stresses are $+1,100$ and $-2,300$; and its length is 5 ft. or 60 ins. The minimum section thought advisable to use consists of 2 angles $2\frac{1}{2}\times2\times\frac{1}{4}$ ins. $=2.12$ sq.ins., with the $2\frac{1}{2}$ in. legs back to back. The least radius of gyration is found to be 0.78 in.; thus $\frac{l}{r}=\frac{60}{0.78}=77$, corresponding, by Table I, to a permissible unit-stress of 9,020 lbs. per sq.in.

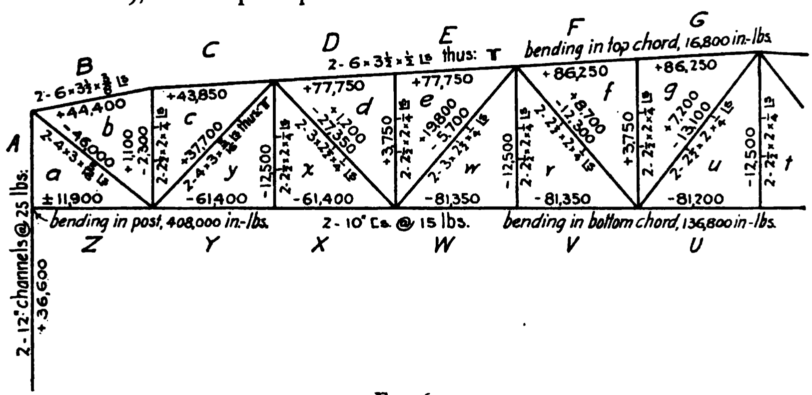


FIG. 64.

In member *cy* the stress is $+37,700$, and its length is 88 ins. Assuming 2 angles $4\times3\times\frac{5}{16}$ in. $=4.18$ sq.ins.; least $r=1.27$ ins.; and $\frac{l}{r}=\frac{88}{1.27}=69$, which corresponds to a permissible unit-stress of 9,490 lbs. per sq.in. Thus the value of this member in compression $=$ $9,490\times4.18=39,700$ lbs.

In member *xd*, the stresses are $+1200$ and $-27,350$. Here the tensile stress, which is much the greater, naturally governs. Then, $27,350\div16,000=1.71$ sq.ins. required. Section provided:

Two angles $3\times2\frac{1}{2}\times\frac{1}{4}$ ins.	$=2.62$
Less four holes $\frac{7}{8}\times\frac{1}{4}$ ins.	$=0.87$
Net area	$=1.75$ sq.ins.

In member *ew*, the stresses are $+19,800$ and $-5,700$ and its length $=93$ ins. Assuming 2 angles $3\times2\frac{1}{2}\times\frac{1}{4}$ ins. $=2.62$ sq.ins.; least

$r=0.95$ in.; and $\frac{l}{r}=\frac{93}{0.95}=98$, which corresponds to a permissible unit-stress of 7,820 lbs. per sq.in. The value of this section in compression is, therefore, $7{,}820\times 2.62=20{,}500$ lbs.

In member vf, the stresses are $+8{,}700$ and $-12{,}300$, and its length $=93$ ins. Assuming 2 angles $2\frac{1}{2}\times 2\times\frac{1}{4}$ ins. $=2.12$ sq.ins.; least $r=0.78$ in.; and $\frac{l}{r}=\frac{93}{0.78}=119$, corresponding to a permissible unit-stress of 6,720 lbs. per sq.in. The value of this section in compression $=6{,}720\times 2.12=14{,}200$. Its net area, allowing for two holes $=1.68$ sq.ins.; thus its value in tension $=1.68\times 16{,}000=27{,}000$ lbs.

Two angles $2\frac{1}{2}\times 2\times\frac{1}{4}$ ins. are also used for the diagonal member gu, as well as for all of the verticals.

Top Chord. In the top chord members Ff and Gg, the direct or axial stress is $+86{,}250$, and the bending moment at adjacent panel-points, 16,800 in.-lbs. This bending moment induces compression in the fibres below the neutral axis; and the section employed must be such that the maximum intensity of compression at the extreme lower fibres, due to the combination of the direct and the bending stresses, shall not exceed 12,000 lbs. per sq.in. Assuming 2 angles $6\times 3\frac{1}{2}\times\frac{1}{2}$ ins., with the longer legs vertical, their combined area $=9.00$ sq.ins., and their section modulus $=8.46$; then

$$\begin{aligned}86{,}250\div 9.00 &= 9{,}600\\ 16{,}800\div 8.46 &= 2{,}000\\ \hline\end{aligned}$$

Thus total stress on extreme lower fibres $=11{,}600$ lbs. per sq.in.

This section is used throughout, except in the end panels, to avoid unnecessary splicing.

In the end member Bb, the direct stress is $+44{,}400$, and the bending as before, 16,800 in.-lbs. Assuming 2 angles $6\times 3\frac{1}{2}\times\frac{3}{8}$ in., area $=6.84$ sq.ins., and $S=6.50$; then

$$\begin{aligned}44{,}400\div 6.84 &= 6{,}500\\ 16{,}800\div 6.50 &= 2{,}600\\ \hline\end{aligned}$$

Thus total stress on extreme lower fibres $=9{,}100$ lbs. per sq.in.

This section is somewhat heavier than is required, but is used because it conforms in general dimensions with that of the central portion of the top chord.

Bottom Chord. In the bottom chord, the maximum direct stress is $-81,350$, and the bending moment at the middle point of any panel, 136,800 in.-lbs. For this case two channels should be employed, as they make a very suitable track for the trolley and are efficient in bending. Their size must be such that the maximum fibre-stress shall not exceed 16,000 lbs. per sq.in. Assuming two channels, $10\times2.6\times0.24$ ins. at 15 lbs. per foot, their gross area (from Carnegie) $=4.46\times2=8.92$ sq.ins., and their moment of inertia $=66.9\times2=133.8$. Allowance must be made for two $\frac{7}{8}$-in. rivet holes in the web of each channel, spaced 3 ins. above and below the centre line. The area of the holes to be deducted $=4\times\frac{7}{8}\times0.24=0.84$ sq.ins.; thus the net area of the channels $=8.92-0.84=8.08$ sq.ins. The moment of inertia of these holes about the neutral axis of the channels $=0.84\times3^2=7.6$; so the net I for channels $=133.8-7.6=126.2$; and, since $S=\dfrac{I}{y_1}$ (Art. 11), their net section modulus $=\dfrac{126.2}{5}=25.2$. Then .

$$\begin{aligned} 81,350\div8.08&=10,050\\ 136,800\div25.2&=5,430 \\ \hline \end{aligned}$$

Thus the total stress on extreme fibres $=15,480$ lbs. per sq.in.

Now it will be assumed that it is necessary to splice the bottom chord in the middle of panel Xx. Here the direct stress is $-61,400$, and the bending moment, 136,800 in.-lbs., as before. In addition to the rivet holes in the webs, one hole must be allowed for in each flange which, at the rivet line, is 0.42 in. thick. The area of the four holes $=4\times\frac{7}{8}\times0.42=1.47$ sq.ins., and the distance from the neutral axis of the channels to the centre of these holes $=5-0.21=4.79$ ins. Thus the net area of the channels $=8.92-(0.84+1.47)=6.61$ sq.ins.; their net amount of inertia

$$\begin{array}{lrr} & =66.9\times2 & =133.8\\ -0.84\times3^2= & 7.6 & \\ -1.47\times4.79^2= & 33.7 & -\ 41.3\\ \hline & & 92.5 \end{array}$$

and their net section modulus $=\dfrac{92.5}{5}=18.5$. Then

$$\begin{aligned} 61,400\div6.61&=9,300\\ 136,800\div18.5&=7,400 \\ \hline \end{aligned}$$

Thus the total stress on extreme fibres $=16,700$ lbs. per sq.in.

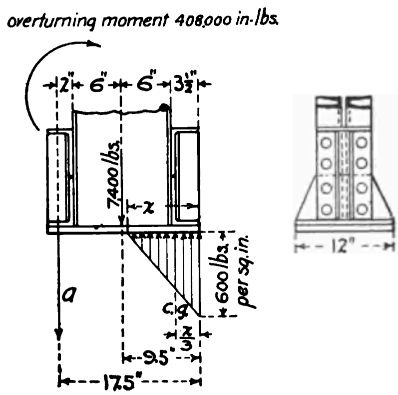
This slightly exceeds the permissible unit-stress, but may be allowed to pass.

Posts. The maximum load on the posts $=36,600$ lbs.; and the bending moment at the bottom chord connection, as well as at the base, $=3400 \times 10=34,000$ ft.-lbs. $=408,000$ in.-lbs. Since the posts are supported laterally in their weak direction, 12,000 lbs. per sq.in. may be taken as the permissible fibre-stress. Assuming two 12-in. channels at 25 lbs. per foot, their area $=14.7$ sq.ins., and their section modulus $=48$. Then

$$36,600 \div 14.7 = 2,500$$
$$408,000 \div 48.0 = 8,500$$

Thus the total stress on the extreme fibres $=11,000$ lbs. per sq.in.

Anchorage for Posts. The bending moment at the base of each post, as determined above $=408,000$ in.-lbs., and is resisted by the counter moments of the load on the post and of the pull on the windward anchor bolt, acting with their individual lever arms about the centre of pressure on the foundation. Assuming a maximum permissible intensity of bearing on concrete of 600 lbs. per sq.in., the total load in Fig. 65 is represented by the area of the shaded triangle



Anchorage for Posts

FIG. 65.

multiplied by the width of the base plate, $=\dfrac{600x}{2} \times 12 = 3600x$. The minimum load acting on the centre-line of the post is the dead-load reaction of the truss, minus the vertical wind-load reaction $=9,000-1,600$

$=7,400$ lbs., as shown; and this load, plus the pull on the anchor bolt a, must be equal to the total load on the foundation; or

$$7,400+a=3,600x$$

from which

$$a=3,600x-7,400. \qquad (1)$$

Now the centre of pressure is $\frac{1}{3}x$ from the edge of the base-plate; thus the lever-arms of the forces which resist the overturning moment are equal to the distances of these forces form the leeward edge of the base-plate, minus $\frac{1}{3}x$; or the lever-arm of the load 7,400 lbs. $=9.5$ ins. $-\frac{x}{3}$, and that of the anchor bolt $a=17.5$ ins. $-\frac{x}{3}$. Then

$$7,400\left(9.5-\frac{x}{3}\right)+a\left(17.5-\frac{x}{3}\right)=408,000. \qquad (2)$$

If the value of a in equation (1) be inserted in equation (2), the latter becomes

$$7,400\left(9.5-\frac{x}{3}\right)+(3,600x-7,400)\left(17.5-\frac{x}{3}\right)=408,000,$$

from which

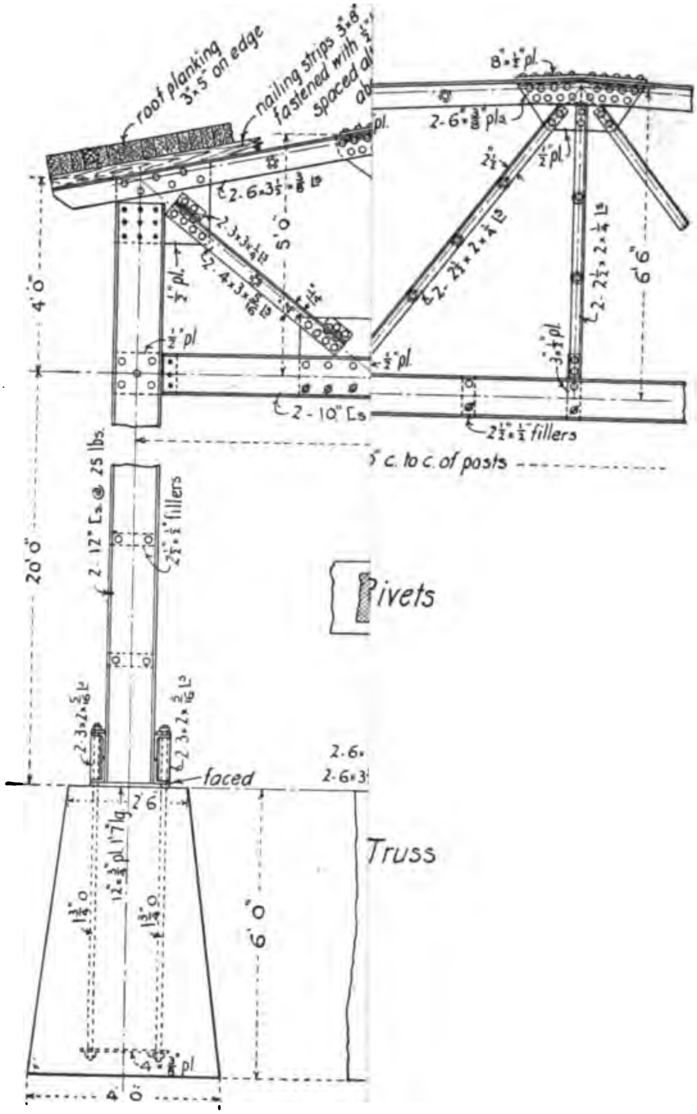
$$-1200x^2+63,000x=467,200$$
$$x^2-52.5x=-389$$
$$x=8.93 \text{ ins.}$$

Consequently $\quad a=(3,600\times 8.93)-7,400=24,750$ lbs.

This result may be checked by taking moments, about the centre of pressure on the foundation (which is $\frac{1}{3}x=2.97$ ins. from the edge of the base-plate), of the load on the post and of the computed tension on the anchor bolts, thus:

$$7,400\times(9.5-2.97) = 48,300$$
$$24,750\times(17.5-2.97)=359,600$$
$$\overline{407,900 \text{ in.-lbs.,}}$$

which is the resisting moment, the slight discrepancy between it and the overturning moment being due to slight inaccuracies in the calculations.



roof planking 3"×5" on edge
8"×½" pl.
2-6"×3/8" pls.
½" pl.
2-6×3½×3/8 Ls
2-3×3×¼ Ls
2-4×3×5/16 Ls
½" pl.
5'0"
4'0"
6'6"
2-10" [s
2½"×½" fillers
2-12" [s @ 25 lbs.
2½"×½" fillers
20'0"
faced
2'6"
6'0"
4'0"
Truss

The anchor-bolts, which are of steel, may be stressed to 16,000 lbs. per sq.in. Then 24,750 ÷ 16,000 = 1.54 sq.ins. sectional area required. Round bars 1¾ ins. diameter are used, their net sectional area at the root of the thread being 1.75 sq.ins.

Details. The details of one-half of a truss and supporting column are shown in Fig. 66. The connection or gusset plates are all ½ in. thick, and the rivets used are ¾ in. diameter. From Table V, Art. 19, the value of a shop-driven rivet in single shear is found to be 5,300 lbs., and in bearing on ½ in. plate, 9,000 lbs. By Table VI, the value of a field-driven rivet in single shear = 3,975 lbs., and in bearing on ½ in. plate = 6,750 lbs.

Referring to Fig. 64, the stress in member *ab* is found to be −46,000; then 46,000 ÷ 9000 = 5.1 rivets required in bearing on ½ in. plate. The detail drawing shows 6 rivets, four of which are in the main angles and two in the lock-angles. In member *cy*, the number of rivets required = 37,700 ÷ 9,000 = 4.2; whereas 5 rivets are used, two being in the lock-angles. In members *xd* and *cw* 3 rivets are used; and in each of the remaining members 2 rivets.

At the centre of the truss, the top-chord is spliced by two 6×⅜ in. plates on the vertical legs of the angles in addition to the ½-in. gusset plate; thus the rivets are in full double shear. The horizontal legs of the angles are spliced by an 8×½ in. plate, the rivets being in single shear. Then the value of the rivets in this splice is as follows:

8 rivets in double shear at 10,600 = 84,800
10 rivets in single shear at 5,300 = 53,000

137,800 lbs.

This is in excess of the direct stress in the adjacent panels, but it must be remembered that there is also a secondary bending moment to provide for.

Bottom-Chord Splice. The bottom-chord is spliced at the centre of panel Xx, where the direct stress = −61,400 lbs., and the bending stress = 136,800 in.-lbs. Now the splice-plates must be of sufficient area so that the maximum stress on the extreme lower fibres of the bottom flange plate, due to the combined stresses, shall not exceed 16,000 lbs. per sq.in.; and the rivets must be so arranged that the stress on those in the bottom flange shall not exceed their permissible single shearing value.

In dealing with a problem of this nature, it is usually most convenient first to design the splice and afterwards to investigate its ability to resist the applied forces. If found to be either too weak or too strong, it may then be re-designed and its value computed anew.

In Fig. 67 is shown an enlarged detail of the splice, whch is composed of a vertical plate $10\times\frac{1}{2}$ in. between the channels, and of two horizontal plates $6\times\frac{3}{8}$ in. on the top and bottom flanges. Now the direct stress induces tension on all of the splice material; whereas the bending moment causes tension in the fibres below the neutral axis only, which tension attains its maximum intensity at the extreme bottom fibres of the flange splice, and must be added to that of the direct stress. Above the neutral axis, the bending moment tends to induce compression, and thus partially counteracts the direct tension.

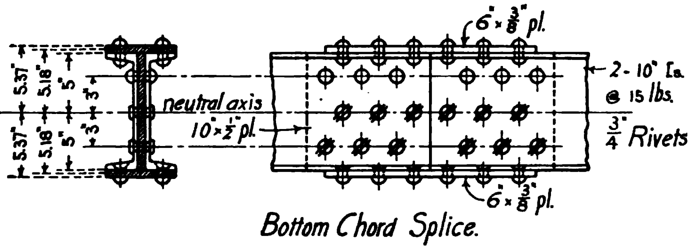


Fig. 67.

In computing the net area of the splice-plates, three holes $\frac{7}{8}$ in. diameter will be deducted from the vertical plate, and two holes $\frac{7}{8}$ in. diameter from each of the horizontal plates, thus:

		Gross Area.		Net Area.
One plate $10\times\frac{1}{2}$ in.	=	5.00	=	3.69
Two plates $6\times\frac{3}{8}$ in.	=	4.50	=	3.19
		9.50 sq.ins.		6.88 sq.ins.

To determine the net section modulus S of the splice-plates, their moment of inertia about the neutral axis is first computed, from which is deducted the moment of inertia of the rivet holes about the same axis; then this result is divided by the distance from the neutral axis to the outer fibres.

Net Moment of Inertia of Splice-Plates about Neutral Axis.

For $10 \times \frac{1}{2}$ in. plate,	$I = \frac{bh^3}{12} = \frac{0.5 \times 10^3}{12}$	$= 41.7$	
For two $6 \times \frac{3}{8}$ in. plates,	$I = 4.5$ sq.ins. $\times 5.18^2$	$= 120.5$	162.2
For two holes 3 ins. from axis,	$I = 0.87$ sq.ins. $\times 3^2$	$= 7.8$	
For four holes 5.18 ins. from axis,	$I = 1.31$ sq.ins. $\times 5.18^2$	$= 35.2$	43.0
			119.2

and

$$S = \frac{I}{y_1} = \frac{119.2}{5.37} = 22.2.$$

Then

61,400 lbs. $\div 6.88 =$ 8,940 lbs. per sq.in. uniform tension from direct stress.

136,800 in.-lbs. $\div 22.2 =$ 6,160 lbs. per sq.in. on bottom fibres from bending;

Total $= 15,100$ lbs. per sq.in. on bottom fibres.

The rivets in the splice may be treated similarly to the plates. Thus the load per rivet due to the direct stress is obtained by dividing this stress by the total number of rivets in one end of the splice, counting each rivet in the vertical plate as two since they are in double shear. There are $9 \times 2 = 18$ rivets in the vertical plate and 6 rivets in each of the two flange plates, making 30 in all. The maximum load per rivet due to bending is obtained by dividing the bending moment by the section modulus S of the rivets; and this section modulus of the rivets may be computed by calculating their moment of inertia about the neutral axis, and dividing by the distance from this axis to the outer rivets. In calculating the moment of inertia of the rivets, each rivet in the web splice is taken as two as before on account of their being in double shear.

Moment of Inertia of Splice Rivets about the Neutral Axis.

$$12 \text{ rivets} \times 3 \text{ ins.}^2 = 108$$
$$12 \text{ rivets} \times 5 \text{ ins.}^2 = 300$$
$$408$$

$$S = \frac{I}{y_1} = \frac{408}{5} = 81.6$$

Then

$$
\begin{aligned}
61{,}400 \div 30 &= 2{,}050 \text{ lbs. per rivet from direct stress;}\\
136{,}800 \div 81.6 &= \underline{1{,}680} \text{ lbs. per rivet in bottom flange from moment;}\\
\text{Total } &= 3{,}730 \text{ lbs. per rivet in bottom flange.}
\end{aligned}
$$

This total load per rivet in the bottom flange splice is less than their single shearing value, viz., 5,300 lbs., and thus the splice is of ample strength.

On account of the trolley which is assumed to run on the bottom-chord channels, the rivets on the upper side of the lower flanges are countersunk, and the two lower rows in the web are flattened to $\frac{1}{4}$ in high. The upper row of rivet heads will clear the trolley wheels.

At the top of the main posts there is a vertical load on the connecting rivets of 36,600 lbs., as shown in Fig. 64. In addition to this vertical load, there is a horizontal force of 8,500 lbs. to be resisted, as shown in Fig. 63(a); which horizontal force, as already explained, is induced by the posts in resisting the bending moment at the bottom-chord connection, caused by the wind-load. Thus the resultant load on the rivets at the top of the posts may be represented by the hypothenuse of a right angle triangle whose vertical and horizontal sides are equal respectively to 36,600 lbs. and 8,500 lbs.; or, the total load on the rivets $=\sqrt{36{,}600^2+8{,}500^2}=37{,}600$ lbs. Since these are field-driven rivets, the number required $=37{,}600\div 6{,}750=6$, whereas the drawing shows 9.

At the connections of the bottom-chord of the truss with the main posts, there is a horizontal load on the connecting rivets equal to the stress in member Za, viz., $\pm 11{,}900$ lbs., as shown in Fig. 64. When the trolley is at one end of the truss, there will also be a vertical load on the rivets of this connection equal to the trolley-load plus 25 per cent for impact $=12{,}500$ lbs.; and the total load on the rivets will be equal to the resultant of these horizontal and vertical loads $=\sqrt{11{,}900^2+12{,}500^2}$ $=17{,}250$ lbs. Then $17{,}250\div 6{,}750=2.56$ field rivets required in bearing on the $\frac{1}{2}$ in. connection-plate, whereas 3 rivets are provided.

At the base of the main posts, the dimensions of the bearing plate have already been determined in connection with the calculations for the anchor-bolts. The brackets are designed to resist the maximum pull on these anchor-bolts, which was found to be 24,750 lbs.; then $24{,}750\div 5{,}300=4.7$ shop-driven rivets in single shear requred, whereas the drawing, Fig. 66, shows 8. The anchor-bolts are made long

enough to extend nearly to the bottom of the piers, and are provided with nuts at their upper and lower extremities. The $4 \times \frac{3}{8}$ in. plate is principally useful during construction of the piers in helping to keep the anchor-bolts in their proper position.

Stability of Piers. The piers must be designed to resist the overturning moment due to the wind force, as well as to support the direct load applied through the posts. The resistance to overturning depends upon the vertical and horizontal bearing values of the soil, and upon the weight of the piers together with the minimum load which they carry. The vertical friction of the soil on the sides of the piers, as well as a certain amount of lateral support contributed by the adjacent foundation walls, increase the resistance to overturning. It is thus difficult, if not impossible, to determine with great accuracy the stability of the piers; but it is believed that the following investigation will lead to satisfactory and safe results.

The dimensions of the trial pier are as follows: Height from bottom of pier to base of post, 6 ft.; length on top, 2.5 ft.; length on bottom, 4 ft.; width throughout, 2 ft.; as shown in Fig. 68. Thus the pier contains 39 cu.ft. of concrete, which, at 130 lbs. per cu.ft., weighs 5,070 lbs. The load on the post from the roof = 9,000 lbs., and the uplift due to the wind force = 1,600 lbs. The pier also supports a part of the 12-inch brick curtain wall, viz., $2 \times 1 \times 24 = 48$ cu.ft., which, at 120 lbs. per cu.ft. = 5,760 lbs. Therefore, the *minimum* total load acting through the centre line of the pier is as follows:

Pier	+5,070
Roof	+9,000
Wind	−1,600
Curtain wall	+5,760
Total	18,230 lbs.

as shown. The horizontal wind force of 3,400 lbs. is assumed to be applied to the post at the point of contraflexure, which is 10 ft. above the top of the pier.

It is found that reasonably well packed soil is capable of resisting with safety a horizontal pressure of about 1,000 lbs. per sq.ft. for each foot below the surface of the ground; but this working pressure should not exceed 8,000 lbs. per sq.ft. The wind force of 3,400 lbs. tends to overturn the pier, thus bringing into action the horizontal

resistance of the soil. On the leeward side of the pier, this horizontal resistance acts in the opposite direction to that of the applied force, and increases from zero at the ground surface to a maximum at a certain distance below the surface; after which it decreases and again becomes zero at the elevation where the horizontal resistance on the windward side begins to act. On the windward side of the pier, the horizontal resistance acts in the same direction as that of the applied

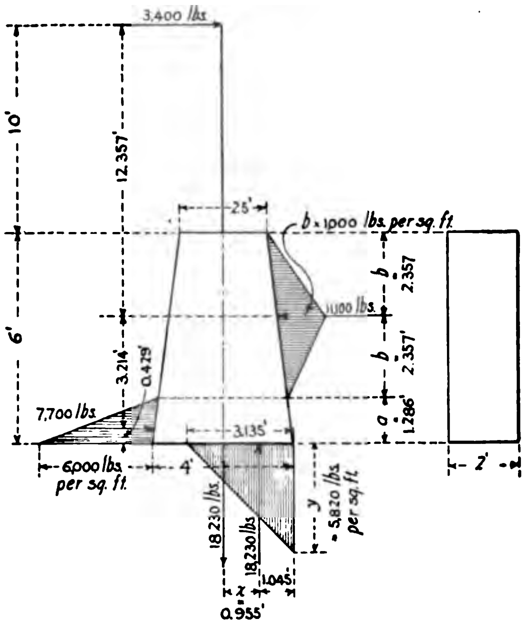


FIG. 68.—Diagram showing Forces acting on Pier.

force, and increases from zero, at the elevation below the surface where the horizontal force on the leeward side ceases, to a maximum at the base of the pier, as shown. For equilibrium, it is necessary that the sum of the forces acting upon a body in one direction shall be equal to the sum of the forces acting upon it in the opposite direction; thus the horizontal resistance of the soil on the leeward side of the pier must be equal to that on the windward side, plus the applied force of 3,400 lbs.

Representing the vertical height of the pressure area on the windward side by *a*, and the vertical height from the upper extremity of

this area to the point of maximum pressure on the leeward side, as well as the vertical height from this point to the ground surface, by b, the following equation may be written:

$$a+2b=6. \qquad (3)$$

Since the base of the pier is 6 ft. below the ground surface, the permissible horizontal pressure at this point is 6,000 lbs. per sq.ft. On the leeward side, the point of maximum pressure is b ft. from the surface; thus the permissible horizontal pressure is here $b\times 1{,}000$ lbs. per sq.ft. Keeping in mind the fact that the pier is 2 ft. wide, another equation may be written, as follows:

$$\frac{2\times 6{,}000a}{2}+3{,}400=2\times 1{,}000b^2,$$

which, when reduced, becomes

$$30a+17=10b^2. \qquad (4)$$

One of the unknown quantities a may be eliminated by multiplying equation (3) by 30 and subtracting from equation (4), thus

$$\begin{array}{rl} 30a+17 &= 10b^2 \\ 30a-180 &= -60b \\ \hline 197 &= 10b^2+60b \\ b^2+6b &= 19.7 \end{array}$$

(3² added to both sides) $b^2+6b+3^2=19.7+3^2=28.7$

(sq. root of above) $b+3=\sqrt{28.7}=\pm 5.357.$

Consequently $b=5.357-3=2.357$ ft.

and $a=6-2b=1.286$ ft.

The pressure area on the windward side $=\dfrac{2\times 6{,}000\times 1.286}{2}=7{,}700$ lbs., and its centre of pressure is located $\frac{1}{3}a$ from the base, $=0.429$ ft., as shown. The pressure area on the leeward side $=2\times 1{,}000\times 2.357^2$ $=11{,}100$ lbs., and its centre of pressure is 2.357 ft. from the top of the pier. Thus the horizontal resistance on the windward side plus the applied force $=7{,}700+3{,}400=11{,}100$ lbs., which is the same as the horizontal resistance on the leeward side. The vertical distance between these two horizontal resistances $=b+\frac{2}{3}a=3.214$ ft., and the

vertical distance from the applied force to the horizontal resistance on the leeward side $=10+b=12.357$ ft.

Now the moment of the wind force, about the point of the leeward horizontal resistance, is balanced by the moment of the windward horizontal resistance about the same point, plus the moment of the minimum total load (which acts through the centre of the pier) about the centre of vertical pressure on the foundation. Representing the distance from the centre of the pier to the centre of vertical pressure on the foundation by x, these moments may be equated thus,

$$18{,}230x+(7{,}700\times 3.214)=3{,}400\times 12.375,$$

from which

$$x=0.955 \text{ ft.}$$

It only remains to find the maximum intensity of pressure on the leeward edge of the foundation. The distance from the centre of pressure to the edge of the pier is equal to one-half the length of the pier, minus x, $=2.0-0.955=1.045$ ft. Since the pressure area is a triangle, its base $=3\times 1.045=3.135$ ft.; and if y represent the maximum intensity of pressure per square foot at the edge of the pier, the pier being 2 ft. wide, then

$$\frac{3.135\times 2y}{2}=18{,}230 \text{ lbs.},$$

from which

$$y=5{,}820 \text{ lbs. per sq.ft.}$$

The maximum vertical pressure per square foot on the foundation is considerably less than a hard clay or gravelly soil is capable of sustaining with safety, and thus the pier has ample stability. If additional stability were required, it might be obtained either by lengthening the base or by making the foundation deeper.

The vertical friction of the soil on the sides of the pier, and the lateral support of the adjacent foundation walls have not been considered in the above investigation, as these forces are somewhat indefinite: they may be counted on as additional factors of safety.

Roof. The object of the steeper slope of the roof at the sides of the building is to prevent the backing up of water due to the formation of ice at the eaves. The author has used this device on several buildings, and has found it to work very satisfactorily.

The roof planking is undoubtedly much thicker than necessary to carry with safety the assumed load of 45 lbs. per sq.ft.; but it has been found by experience that planking of less thickness than 5 ins., when used for a span of 15 ft., may deflect too much, and thus injure the roof covering.

For solid timber roof construction generally, it is recommended that 3-inch planking be used for all spans up to and including those of 10 ft.; 4-inch planking for spans over 10 ft., but not exceeding 13 ft.; 5-inch planking for spans over 13 ft., but not exceeding 16 ft. In cases where the roof trusses are a greater distance apart than 16 ft., it is usually advisable to employ longitudinal steel purlins, spaced not more than 13 ft. c. to c., and to lay the roof planking crosswise with the building.

CHAPTER IX

THE DESIGN OF A PLATE-GIRDER

PLATE-GIRDERS are used principally for railway bridges, but are also employed very largely in warehouses and other buildings. Although in construction a plate-girder is comparatively simple, the stresses therein are somewhat complex; and there are many points in its design requiring careful attention, the greater number of which will be dealt with in the following example:

Data for Design.

Length, c. to c. of end bearings, 50 ft.

Depth, back to back of flange angles, 5 ft. $\frac{1}{2}$ in.

Total load, including weight of girder, 5,000 lbs. per lin.ft., assumed to be applied to the top flange.

Top flange to be supported laterally at intervals not exceeding twelve times its width.

Ends of girder to rest on walls of Portland cement concrete, not less than 24 ins. thick.

Unit-stresses as given in Art. 18, Chap. V.

Rivets, $\frac{7}{8}$ in. diameter.

Thickness of Web-Plate. The web-plate must have a sectional area great enough to resist the maximum shear, and it must be thick enough to give sufficient bearing for the rivets in the flange angles and end stiffeners. Unless otherwise specified, the permissible unit shear for web-plates is usually understood to be on the gross area. In the present case it is 10,000 lbs. per sq.in.

The maximum shear at either end of the girder is equal to $\frac{wl}{2}$ (Art. 6, Chapter II) $=\frac{5{,}000\times 50}{2}=125{,}000$ lbs., which is one-half of the total load. Then $125{,}000\div 10{,}000=12.5$ sq.ins. sectional area required in web-plate to resist the vertical shear. Thus a $60\times\frac{5}{16}$ in. web-plate, which has a sectional area of 18.75 sq.ins. (being of the minimum

thickness allowed in ordinary practice) will be used, provided it subsequently proves to be thick enough for rivet bearing.

Flanges. The maximum bending moment, which is at the centre of the span, is given by the formula $M=\frac{wl^2}{8}$ (Art. 6) $=\frac{5,000\times 50^2}{8}=$ 1,562,500 ft.-lbs.; and the flange stress at the centre is equal to this moment divided by the effective depth of the girder.

The effective depth of a plate-girder is the distance c. to c. of gravity of the flanges; therefore it cannot be known exactly before the section of the flanges has been decided on. When the flanges are composed of two angles and cover-plates, it will usually be found that the effective depth is approximately equal to the depth back to back of angles; so this depth may be used in preliminary calculations to determine the flange section. The centre of gravity of the flanges can then be readily computed, and the correct effective depth thus obtained. If this effective depth is found to be appreciably less than the distance back to back of angles, the calculations should be corrected accordingly, but the effective depth of a plate-girder is never assumed to be greater than the distance back to back of flange angles.

Bottom Flange. Returning to the example, the effective depth will be assumed as 5 ft. Then the flange stress at the centre of the girder $=$ 1,562,500 ft.-lbs. $\div 5 =$ 312,500 lbs.; and the net sectional area required for the bottom flange $=312,500\div 16,000=19.53$ sq.ins.

It is frequently specified that the flanges of plate-girders shall have sufficient sectional area to resist the entire bending moment, taking no account of the bending resistance of the web-plate. But the more general and better practice at the present day is to make due allowance for this bending resistance of the web-plate as follows:

It has been shown in Art. 7 that the section modulus S of a beam of rectangular cross-section

$$=\frac{bh^2}{6}=\frac{bh}{6}h=\frac{A}{6}h, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which b = the breadth of the beam in inches;

h = the height of the beam in inches;

$A=bh$ = the sectional area of the beam in square inches.

Now the web-plate is a rectangle, and, if allowance be made for vertical lines of rivet holes 1 in. in diameter and 4 ins. centre to centre,

its net sectional area becomes three-fourths of its gross area; thus its net section modulus

$$=\frac{3}{4}\cdot\frac{A}{6}h=\frac{A}{8}h. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

For the flanges, the net section modulus may be represented by

Fh, in which F = the net sectional area of one flange, in square inches,
h = the distance centre to centre of gravity of flanges in ins.

Then, since h is practically equal to the height of the web-plate, the total net section modulus of the girder

$$=\frac{A}{8}h+Fh=\left(\frac{A}{8}+F\right)h. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

That is to say, the total net section modulus is equal to one-eighth of the sectional area of the web-plate, plus the net sectional area of one flange, multiplied by the distance centre to centre of gravity of the flanges; or one-eighth of the gross sectional area of the web-plate may be considered as equivalent flange area for the tension flange.

The sectional area of the angles should, when practicable, be equal to at least one-third of the total flange area; and it is desirable that the metal of the angles and of the several flange plates should be of nearly equal thicknesses.

In determining the net sectional area of the flanges, the rivet holes should be assumed $\frac{1}{8}$ in. larger than the diameter of the rivets before driving. In general, when there are rivet holes in both legs of an angle, or if there are two lines of holes in one leg, staggered 2 inches or less, two holes should be allowed for. If there is but a single line of rivet holes in one leg of an angle, or if there are two lines staggered more than 2 inches, then it is only necessary to allow for one hole.

The sectional area required for the bottom flange at the centre of the girder, as already determined, = 19.53 sq.ins., which may be provided for as follows:

One-eighth of $60\times\frac{5}{16}$ in. web-plate	= 2.34
Two angles $6\times4\times\frac{1}{2}$ in.	= 7.50 (4 holes $1\times\frac{1}{2}$ in.)
One plate $13\times\frac{1}{2}$ in.	= 5.50 (2 holes $1\times\frac{1}{2}$ in.)
One plate $13\times\frac{7}{16}$ in.	= 4.81 (2 holes $1\times\frac{7}{16}$ in.)
	20.15 sq.ins. net area.

In accordance with common practice, the thinner plate will be placed furthest from the angles. It will be found that the centre of gravity of this section is about 0.15 in. inside of the angles; but, since the distance back to back of flange angles is ½ in. greater than the height of the web-plate, the assumed effective depth is within the actual, and will thus be retained in the calculations.

Lengths of Cover-Plates. In Art. 6, it has been shown that the bending moment at any point of a beam supported at both ends and uniformly loaded may be represented by a parabola, having its vertex at the centre of the span, and its depth at this point (measured from the horizontal closing line) equal to the centre moment. Now, since the required flange area at any point is proportional to the moment at that point, a parabola may also be drawn so as to represent the required areas, as follows:

Fig. 69 (*a*) represents one-half of the girder, divided into equal panels of 5 ft., the panel-points being denoted by the letters *a* to *f*. Below the girder, in Fig. 69 (*b*), a semi-segment of a parabola is constructed with its base equal to one-half of the span, =25 ft.; and its depth at the centre, equal to the flange area provided at that point, =20.15 sq.ins. The horizontal line *a* 0–*f* 0 represents the base, which is divided into five equal parts by vertical lines through the panel-points of the girder. The vertical line *a* 0–5 at the end of the girder, which is equal to the vertical line *f* 0–*f* 5 at the centre, is made to represent by scale 20.15 sq.ins., and is divided into the same number of equal parts as the base, the points of division being denoted by the figures 0 to 5. From these points, lines are drawn to the point *f* 5, which is the vertex of the parabola. Then the intersections of the verticals through the panel-points with the radial lines through the points 0 to 5 are points on the parabola. Thus, the intersection of the vertical through *b* with the radial line through 1 gives the point *b* 1; the intersection of the vertical through *c* with the radial line through 2 gives the point *c* 2; the intersection of the vertical through *d* with the radial line through 3 gives the point *d* 3; and the intersection of the vertical through *e* with the radial line through 4 gives the point *e* 4. Then the parabola passes through the points *a* 0, *b* 1, *c* 2, *d* 3, *e* 4, and *f* 5; and the flange area required at any point is equal to the ordinate, at that point, between the base line *a* 0–*f* 0 and the parabola.

The flange material is then superimposed on the parabola. The first rectangle, representing the equivalent flange area of the web-

plate, and the second rectangle, representing the net area of the two flange angles, extend the full length of the girder. The third rectangle, representing the net area of the $13\times\frac{1}{2}$ in. cover-plate, is required theoretically to the point where the parabola cuts the lower boundary of the second rectangle, its net length being x_1; and the fourth rectangle, representing the net area of the $13\times\frac{7}{16}$ in. cover-plate, is required theoretically to the point where the parabola cuts the lower

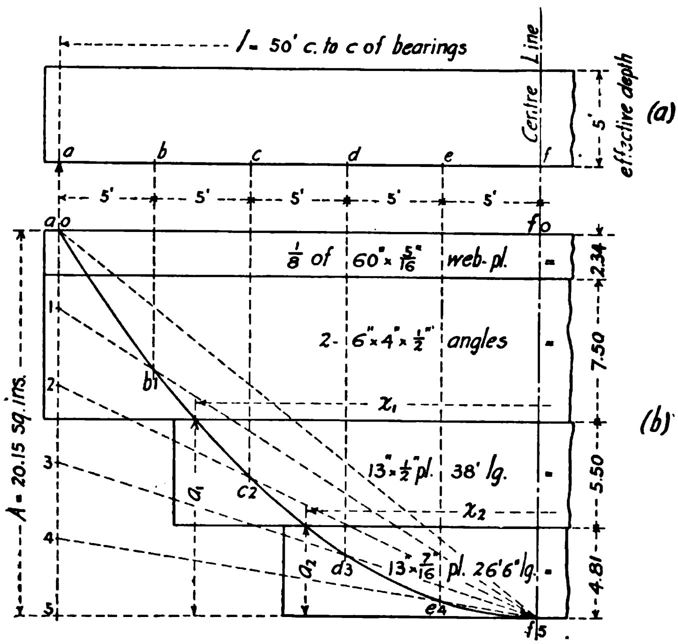


Plate-Girder: Diagram of Flange Material.

FIG. 69.

boundary of the third rectangle, its net length being x_2. Both of the cover-plates, however, are extended about one foot beyond the points to which they are theoretically required in order that enough rivets may be put into the ends of these plates to develop a portion of their value before they are actually required.

Formula for Computing Lengths of Cover-Plates. If desired, the lengths of the cover-plates may be computed analytically by a formula based on one of the laws of the parabola, viz., that the ordinates, measured from a horizontal axis passing through the vertex, vary as

the squares of the corresponding abscissæ, measured from a vertical axis passing through the same point. This formula may be derived as follows:

Referring to Fig. 69 (*b*):

l = length of span, c. to c. of bearings;
x = theoretical length of any cover-plate;
A = total sectional area of flanges at centre of span, = maximum ordinate when $\frac{x}{2}=\frac{l}{2}$;
a = ordinate to parabola at $\frac{x}{2}$.

Then

$$\frac{A}{\left(\frac{l}{2}\right)^2}=\frac{a}{\left(\frac{x}{2}\right)^2},$$

from which

$$x=\sqrt{\frac{al^2}{A}}. \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For the theoretical length x_1 of the $13\times\frac{1}{2}$ in. cover-plate, $l=50$ ft., $A=20.15$ sq.ins., and $a_1=(4.81+5.50)=10.31$ sq.ins.; thus

$$x_1=\sqrt{\frac{10.31\times 50^2}{20.15}}=35.8 \text{ ft.}$$

For the theoretical length x_2 of the $13\times\frac{7}{16}$ in. cover-plate, $a_2=4.81$ sq.ins; and

$$x_2=\sqrt{\frac{4.81\times 50^2}{20.15}}=24.4 \text{ ft.}$$

Both of these calculations may be performed by a single setting of the slide rule. The $13\times\frac{1}{2}$ in. cover-plate should be about 38 ft. long, and the $13\times\frac{7}{16}$ in. cover plate about 26 ft., 6 ins. long, as shown.

Top Flange. Having decided upon the section and lengths of cover-plates for the bottom flange, it is usually customary to make the top flange the same, except in rare cases, in which the top flange is unsupported laterally, when it may be figured as a column of a length equal to one-half of the span. In the example, the top flange

is assumed to be supported laterally at intervals not exceeding twelve times its width; consequently it is made like the bottom.

Rivet Spacing in Flanges. The rivets connecting the web-plate to the flange angles are required to transmit the longitudinal shearing stresses (discussed in Art. 12) from the web-plate to the flanges. Considering any section of the girder, the longitudinal shear per lineal inch at the flanges may be assumed equal to the total vertical shear at the section divided by the effective depth, in inches; and the net amount of this shear to be transmitted by the rivets to the flanges is proportioned to

$$\frac{\text{Net area of one flange}}{\text{Net area of one flange} + \frac{1}{8} \text{ of web-plate section}};$$

for the total equivalent area of the flanges includes one-eighth of the sectional area of the web-plate, as already demonstrated.

When the load is applied directly to the top flange, as assumed in the example, the rivets in this flange are also called upon to transmit this load to the web-plate. Then the total stress per lineal inch to be resisted by the top flange rivets is equal to the resultant of the net longitudinal shear combined with the vertical load.

The required pitch of the flange rivets will be found to increase gradually from the ends towards the centre of the girder; and, consequently, to be greater at one end of a panel than at the other. In the following computations, the rivet pitch is determined at the centre of each panel, which is the average pitch required for the whole panel. By this method the rivets at one end of a panel will be a little too far apart, and those at the other end a little too close together; whereas the total number of rivets in the panel will be correct; but there should be no objection to treating the subject in this manner, provided the panel lengths are not greater than the depth of the girder. Now the vertical shear at any section of the girder is equal to the end reaction, minus the load between this section and the nearer end (Art. 6); and the bearing value of one $\frac{7}{8}$-inch rivet on the $\frac{5}{16}$-inch web-plate (by Table V, Art. 19) $=6,560$ lbs. The rivet spacing in the flanges will now be computed.

Rivet Spacing in Flanges for Panel ab. The vertical shear at the centre of the panel $=125,000-(5,000\times 2.5)=112,500$ lbs.; and, as shown in Fig. 69, the flange section here consists of but two $6\times 4\times \frac{1}{2}$ in. angles, $=7.50$ sq.ins. net area, plus $\frac{1}{8}$ of the $60\times \frac{5}{16}$ in.

web-plate, $=2.34$ sq.ins.; while the effective depth of the girder is found to be 58.5 ins.

Then, the longitudinal shear per linear inch at the centre of gravity of the flanges $=112{,}500 \div 58.5 = 1{,}925$ lbs.; and the longitudinal stress per linear inch on the rivets $=1{,}925 \times \dfrac{7.50}{7.50+2.34} = 1{,}465$ lbs. Therefore, since the value of one rivet is 6,560 lbs., the required spacing for the bottom flange $=6{,}560 \div 1{,}465 = 4.47$ ins.

On the top flange, the vertical load $=5{,}000$ lbs. per lin.ft., $=417$ lbs. per lin.in. Consequently, the total stress per linear inch on the top flange rivets $=\sqrt{1{,}465^2+417^2}=1{,}520$ lbs.; and the required spacing for this flange $=6{,}560 \div 1{,}520 = 4.32$ ins.

Rivet Spacing in Flanges for Panel bc. The vertical shear at the centre of the panel $=125{,}000-(5{,}000 \times 7.5)=87{,}500$ lbs.; the flange section consists of one $13 \times \frac{1}{2}$ in. plate $=5.50$ sq.ins., net area, plus two $6 \times 4 \times \frac{1}{2}$ in. angles, $=7.50$ sq.ins. net area, plus one-eighth of the $60 \times \frac{5}{16}$ in. web-plate, $=2.34$ sq.ins.; and the effective depth becomes 59.5 ins.

Then the longitudinal shear per linear inch at the centre of gravity of the flanges $=87{,}500 \div 59.5 = 1{,}470$ lbs.; and the longitudinal stress per linear inch on the rivets $=1{,}470 \times \dfrac{5.50+7.50}{5.50+7.50+2.34} = 1{,}245$ lbs. Thus the required spacing for the bottom flange $=6{,}560 \div 1{,}245 = 5.26$ ins.

The total stress per linear inch on the top flange rivets

$$=\sqrt{1{,}245^2+417^2}=1{,}315 \text{ lbs.};$$

and the required spacing $=6{,}560 \div 1{,}315 = 5$ ins.

Rivet Spacing in Flanges for Panel cd. The vertical shear at the centre of the panel $=125{,}000-(5{,}000 \times 12.5)=62{,}500$ lbs.; the flange section is the same as for panel *bc*; and the effective depth is also the same.

Then the longitudinal shear per linear inch at the centre of gravity of the flanges $=62{,}500 \div 59.5 = 1{,}050$ lbs.; the longitudinal stress per linear inch on the rivets $=1{,}050 \times \dfrac{5.50+7.50}{5.50+7.50+2.34} = 890$ lbs.; and the required spacing for the bottom flange $=6{,}560 \div 890 = 7.38$ ins.

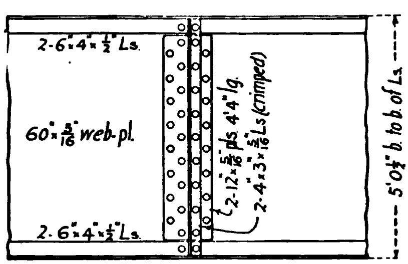
The total stress per linear inch on the top flange rivets $=$

$\sqrt{890^2+417^2}=980$ lbs.; and the required spacing $=6,560\div980=6.70$ inches.

It is unnecessary to proceed further, as the maximum spacing should not exceed 6 ins. In order to simplify shop-work, it is usually customary to make the rivet spacing in the top and bottom flanges alike. Consequently the pitch of the rivets in the vertical legs of the flange angles will be made 4 ins. from *a* to *b*, 5 ins. from *b* to *c*, and 6 ins. from *c* to *f*, as shown in Fig. 74. Since the minimum required spacing is greater than 3 ins., the web-plate is of ample thickness.

Rivet Spacing in Cover-Plates. The first cover-plate requires a sufficient number of rivets between its end and the end of the second cover-plate to develop its full value in this distance; and the value of the second, or outer, cover-plate must likewise be developed between its end and the centre of the span. Now the tension value of the first cover-plate is equal to its net sectional area multiplied by the permissible stress per sq.in. $=5.50\times16,000=88,000$ lbs.; the value of one $\frac{7}{8}$-inch rivet in single shear (by Table V) $=7,220$ lbs.; and the distance between the ends of the first and second cover-plates $=5$ ft. 9 ins. $=69$ ins. Then, the number of rivets required in this distance $=88,000\div7,220=12$; and, since there are two lines of rivets in the plate, the required spacing $=69\div\frac{12}{2}=11\frac{1}{2}$ ins. But practical considerations require that this spacing should not exceed 6 ins., and thus it will be made to stagger with the rivet pitch in the vertical legs of the angles.

Web Splice. When specified that the flanges shall be assumed to resist the total bending moment, and the web-plate, to resist shear only, it is usually customary to design the web splice to sustain the maximum vertical shear at the section, using a pair of splice plates of height equal to the clear distance between the flange angles, and wide enough to contain two lines of rivets each side of the joint, the vertical spacing on a single line not exceeding 6 ins., as shown in Fig. 70.



Web Splice

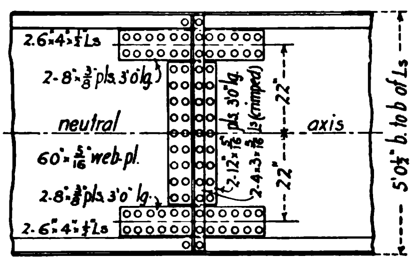
FIG. 70.

Assuming that the web-plate is to be spliced at panel-point *d*,

which is distant 15 ft. from the nearer end support, the vertical shear = 125,000 − (5,000 × 15) = 50,000 lbs.; and, since the bearing value of a $\frac{7}{8}$-inch rivet on a $\frac{5}{16}$-inch plate = 6,560 lbs., the number of rivets required to resist this vertical shear = 50,000 ÷ 6,560 = 8, whereas seventeen rivets are provided. But this splice is unsatisfactory, for the web-plate must take its due proportion of the bending stresses, notwithstanding any assumptions to the contrary; and, if the splice be not as efficient as the web-plate, it is obvious that it will be overstressed.

When the more correct assumption is made, viz., that one-eighth of the sectional area of the web-plate is equivalent to so much flange area, it is customary to design the splice so that its resistance to bending will be equal to that of the web-plate. This may be accomplished (more or less effectually) in various ways, as follows:

Fig. 71 illustrates a form of web splice used by some engineers, in which the horizontal plates adjacent to the flange angles are usually assumed to resist the bending moment, and the vertical plates, the shear.



Web Splice

FIG. 71.

Now the bending value of the web-plate is equal to one-eighth of its sectional area, multiplied by its height and by the permissible maximum fibre stress, = 2.34 × 60 × 16,000 = 2,246,200 in.-lbs.; and the horizontal splice plates, as well as the rivets therein, are to be proportioned so that their resisting moment will be equal to or greater than this bending value. Since the bending stresses in a girder increase uniformly from zero at the neutral axis to a maximum at the outer fibres, the bending stress per square inch at any point is equal to that at the outer fibres, multiplied by the distance from the neutral axis to this point, and divided by one-half of the height of the girder. Assuming the horizontal splice-plates to be 8 ins. wide, the distance from the neutral axis to their centre line is 22 ins., as shown; and the permissible unit-stress at this point = $16,000 \times \frac{22}{30} = 11,740$ lbs. per sq.in. Then the net area of the upper and lower plates, multiplied by this unit-stress, and by the distance from the neutral axis to the centre of these plates, must be equal to the bending value of the

web-plate; or, representing the net area required in one pair of splice-plates by A,

$$2A\times 11{,}740\times 22=2{,}246{,}400 \text{ in.-lbs.},$$

from which

$$A=4.35 \text{ sq.ins.}$$

The net area of the two $8\times\frac{3}{8}$ in. plates used (allowing for two one-inch holes in each) $=4.50$ sq.ins.

The value of a rivet in resisting the bending moment attributed to the web-plate may also be considered to increase uniformly from zero at the neutral axis to a maximum at the top or bottom of the girder. Then, if 6,560 lbs. (the permissible bearing value of a $\frac{7}{8}$-inch rivet on a $\frac{5}{16}$-inch plate) be the value of a rivet at a distance of 30 ins. from the neutral axis, its value at the centre of the horizontal splice-plates (22 ins. from this axis) will be equal to $6{,}560\times\frac{22}{30}=4{,}810$ lbs.; and the number of rivets in the upper and lower plates, multiplied by this value, and by the distance from the neutral axis to the centre of gravity of the rivets, must be equal to the bending value of the web-plate; or, representing the number of rivets required in one pair of splice-plates by n,

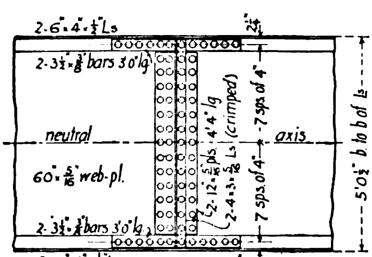
$$2n\times 4{,}810\times 22=2{,}246{,}400 \text{ in.-lbs.},$$

from which

$$n=10.6 \text{ rivets},$$

whereas twelve rivets are used.

This splice may be efficient, but it is not to be recommended; as it concentrates the greater part of the bending stress in the web-plate at two points, thereby inducing unknown internal stresses; besides which, it is awkward and uneconomical.



Web Splice

FIG. 72.

Another form of web splice used is shown in Fig. 72. In this the vertical splice-plates are assumed to resist bending moment to the value of the rivets therein; and the balance of the bending moment is supposed to be resisted by the flats on the vertical legs of the angles.

Now the value of a rivet in resisting bending moment at a distance of one inch from the neutral axis $=6,560 \div 30 = 219$ lbs.; thus the value of a rivet at any point between the neutral axis and the top or bottom of the girder will be equal to 219 lbs. multiplied by the distance, in inches, of the rivet from the axis; and the resisting moment of the rivet will be equal to 219 lbs. multiplied by the square of this distance. Therefore, the total moment of resistance of the rivets in the vertical splice-plates will be equal to the summation of the squares of their several distances from the neutral axis, multiplied by the value of a rivet at a distance of one inch from the axis.

Since the net equivalent flange area of the web-plate is based on a minimum vertical rivet spacing of 4 ins., this pitch is used in the design of the splice, as shown; and the moment of resistance of one vertical row of rivets is as follows:

$$\begin{aligned} 4^2 &= 16 \\ 8^2 &= 64 \\ 12^2 &= 144 \\ 16^2 &= 256 \\ 20^2 &= 400 \\ 24^2 &= 576 \\ \hline & 1,456 \times 2 \times 219 = 638,000 \text{ in.-lbs.} \end{aligned}$$

Consequently, since there are two vertical rows of rivets on either side of the joint, their total moment of resistance $=638,000 \times 2 = 1,276,000$ in.-lbs.; and the difference between this and the bending value of the web-plate ($2,246,400 - 1,276,000 = 970,400$ in.-lbs.) must be resisted by the splice-bars on the vertical legs of the flange angles.

Now the permissible unit-stress at the centre line of these splice-bars $=16,000 \times \frac{28}{30} = 14,940$ lbs. per sq.in.; then, if A = the net area of one pair of bars,

$$2A \times 14,940 \times 28 = 970,400 \text{ in.-lbs.},$$

from which

$$A = 1.58 \text{ sq.ins.};$$

and the net area of the two $3\frac{1}{2} \times \frac{3}{8}$ in. splice-bars used $=1.87$ sq.ins.

The value of one rivet in these bars $=6,560 \times \frac{28}{30} = 6,120$ lbs.; then, if n = the number of rivets required in one pair of bars,

$$2n \times 6,120 \times 28 = 970,400 \text{ in.-lbs.},$$

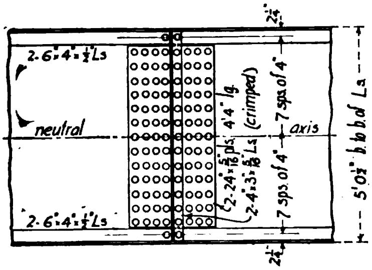
from which

$$n = 3 \text{ rivets.}$$

But the rivets in these bars are also required to transmit longitudinal shear; and, when the load is applied directly to the flange, the rivets therein are further called upon to transmit this load to the web-plate. Thus it is necessary to increase the number of rivets as obtained above; consequently the drawing shows six.

This splice is not altogether satisfactory; as the bars on the vertical legs of the flange angles are of doubtful utility, and they are evidently incapable of transmitting the diagonal stresses of compression and tension mentioned in Art. 12, Chap. II.

A better form of web splice, which is shown in Fig. 73, consists of a pair of plates of a height equal to the clear distance between the flange angles, and wide enough to contain a sufficient number of rivets, uniformly spaced, to resist the entire bending moment attributable to the web-plate.



Web Splice

FIG. 73.

It has been shown in the example, illustrated by Fig. 72, that the moment of resistance of a single vertical row of rivets = 638,000 in.-lbs.; therefore, since the bending value of the web-plate = 2,246,400 in.-lbs., the number of rows required to equal this value = 2,246,400 ÷ 638,000 = 3.5. The drawing shows four vertical rows of rivets each side of the joint, thus requiring splice-plates 24 ins. wide.

Now the outer fibre stress for the splice-plates, which are 52 ins. high, $= 16{,}000 \times \frac{26}{30} = 13{,}860$ lbs. per sq.in.; and the thickness of these plates must be such that their moment of resistance, with this maximum fibre stress, will be equal to or greater than the moment of resistance of the web-plate, viz., 2,246,400 in.-lbs. Then, assuming two $52 \times \frac{5}{16}$ in. splice-plates, their area = 32.5 sq.ins., and their net moment of resistance

$$= \frac{32.5}{8} \times 52 \times 13{,}860 = 2{,}928{,}000 \text{ in.-lbs.}$$

Consequently, the splice-plates are of ample thickness.

The most satisfactory form of web-splice is that shown in Fig. 74, in which a pair of plates are used, wide enough to permit of three vertical rows of rivets, 4 ins. pitch, each side of the joint, and of the full height of the web-plate. These plates are crimped or offset at the ends to fit over the vertical legs of the flange angles. Thus the web-plate is fully spliced at every point, and is capable of resisting all longitudinal and diagonal stresses of tension and compression in exactly the same manner as though it were uncut.

Flange Splice. When practicable, the flange material should be ordered in full lengths, as splices are objectionable, particularly in the angles. For the purpose of illustration, however, one flange angle is spliced 2 ft. either side of the centre, and the first cover-plate, at the centre, as shown in Fig. 74.

Now the net sectional area of one $6\times4\times\frac{1}{2}$ in. angle$=3.75$ sq.ins., and that of one $5\frac{1}{2}\times3\frac{1}{2}\times\frac{5}{8}$ in. splice angle$=3.98$ sq.ins. The value of the flange angle in tension$=3.75\times16{,}000=60{,}000$ lbs., and the value of one $\frac{7}{8}$-inch rivet in single shear$=7{,}220$ lbs.; then $60{,}000\div7{,}220=8.3$ rivets required in either end of the splice angle. These rivets should be disposed between the two legs of the splice angle in proportion to their widths—or six in the $5\frac{1}{2}$-inch leg and three in the $3\frac{1}{2}$-inch leg; but, for convenience, five rivets are used in the latter, as shown. The $5\frac{1}{2}\times3\frac{1}{2}$ in. splice angles are made from $6\times3\frac{1}{2}$ in. angles by planing off the longer leg and rounding the back to fit the fillet of the flange angles.

The net area of the $13\times\frac{1}{2}$ in. cover-plate$=5.50$ sq.ins., and its value in tension$=5.50\times16{,}000=88{,}000$ lbs. The splice-plate is of the same dimensions as this cover-plate, and the number of $\frac{7}{8}$-inch rivets in single shear required$=88{,}000\div7{,}220=12.2$. But, since this splice-plate is separated from the plates to be spliced by the intervening $13\times\frac{7}{16}$ in. cover-plate, the rivets are less effective than they would be if the splice-plate and the plates to be spliced were in direct contact. Thus the specification of the American Railway Engineering and Maintenance of Way Association requires in such cases that the number of rivets shall be increased one-third for each intervening plate. By this rule, the number of rivets in each end of the splice-plate should be $12.2\times1\frac{1}{3}=16.3$; whereas the drawing shows twenty rivets.

End Stiffeners and Bearing Plates. The end stiffeners, which act as columns, receive their load from the web-plate and transmit it to the bearing plates. The maximum end shear, as already computed,

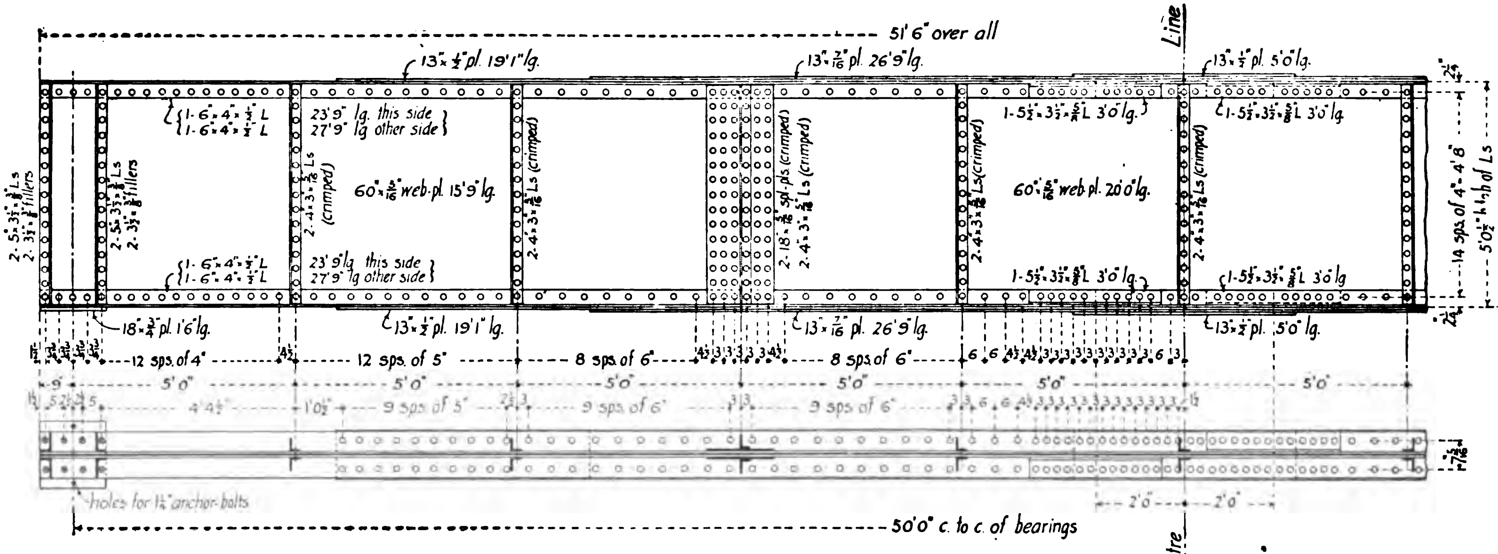


Plate-Girder: Detail Drawing

FIG. 74.

=125,000 lbs.; and, since the ratio of the length of the end stiffeners to their radius of gyration is small, the compression unit-stress of 12,000 lbs. per sq.in. may be used without reduction; thus, 125,000÷12,000=10.42 sq.ins. required. Two pairs of $5\times3\frac{1}{2}\times\frac{3}{8}$ in. angles are used, their total sectional area being equal to 12.20 sq.ins., and, between these angles and the web plate, $3\frac{1}{2}\times\frac{1}{2}$ in. fillers are employed to avoid crimping. These stiffeners should be placed as shown in Fig. 74, and ground or milled to fit perfectly against the bottom flange angles; while the number of rivets required therein is equal to the end shear divided by the bearing value of one $\frac{7}{8}$-inch rivet on the $\frac{5}{16}$-inch web plate, =125,000÷6,560=19. The size of the bearing plates is determined by dividing the reaction or end shear by the permissible bearing value of concrete. Thus 125,000÷400=312.5 sq.ins. required, whereas the area of the plates provided, which are $18\times18\times\frac{3}{4}$ in., =324 sq.ins.

Intermediate Stiffeners. In the case of a heavy concentrated load at any intermediate point of a plate-girder, stiffeners should be provided and proportioned in the same manner as those at the ends, the object of these stiffeners being to transmit this load to the web-plate. Otherwise the intermediate stiffeners are simply to prevent the web-plate from buckling, and they are usually spaced about as far apart as the depth of the girder. There is no scientific method of proportioning them; but the following rule, taken from the specification of the American Railway Engineering and Maintenance of Way Association, will be found to give satisfactory results, viz.: The width of the outstanding legs of intermediate stiffeners should be equal to one-thirtieth of the depth of the girder, plus 2 ins. Thus, for the present example, the depth of the web-plate=60 ins.; and $\frac{60}{30}+2=4$ ins. The stiffeners used consist of $4\times3\times\frac{5}{16}$ in. angles, crimped at the ends to fit over the flange angles.

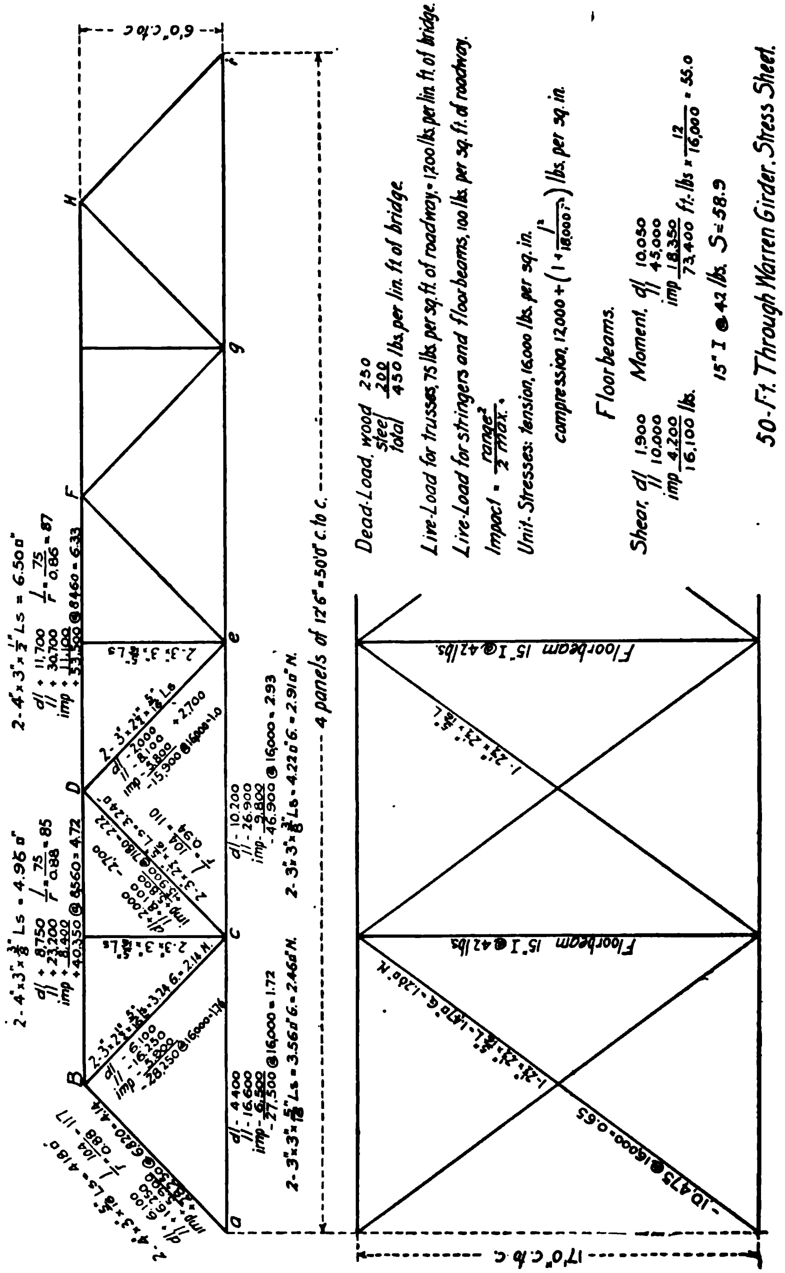

Dead-Load, wood 250
steel 200
total 450 lbs. per lin. ft. of bridge.
Live-Load for trusses, 75 lbs. per sq. ft. of roadway = 1200 lbs. per lin. ft. of bridge.
Live-Load for stringers and floorbeams, 100 lbs. per sq. ft. of roadway.
Impact = range² / 2 max.
Unit-Stresses: tension, 16,000 lbs. per sq. in.
compression, 12,000 + (1 + l²/18,000 r²) lbs. per sq. in.
Floorbeams.
Shear, dl 1,900
ll 10,000
imp 4,200
16,100 lbs.
Moment, dl 10,050
ll 45,000
imp 18,350
73,400 ft.-lbs. × 12/16,000 = 55.0
15" I @ 42 lbs. S = 58.9
Floorbeam 15" I @ 42 lbs.
4 panels of 12'6" = 50'0" c. to c.
17'0" c. to c.
6'0" c. to c.
50-Ft. Through Warren Girder. Stress Sheet.

FIG. 75.

CHAPTER X

THE DESIGN OF A 50-FOOT THROUGH WARREN GIRDER HIGHWAY BRIDGE

THIS type of bridge is particularly suitable for spans of from 25 to 75 ft. The top chords, which are not directly braced in a horizontal plane, depend upon the rigidly-connected floorbeams for lateral support; and thus it is advisable to use comparatively short panels. The structure considered in this chapter is intended for service on an ordinary country road, and will be designed in accordance with the following data:

Length, c. to c. of bearings, 50 ft., =4 panels of 12 ft. 6. ins.
Depth, c. to c. of chords, 6 ft.
Length of diagonal web-members $=\sqrt{6^2+6.25^2}=8.67$ ft.
Width, c. to c. of trusses, 17 ft.
Roadway, 16 ft. clear.
Floor planking, spruce or pine, 3 ins. thick.
Stringers, spruce or pine, 3×12 ins., about 2 ft. apart.
Wheel-guards, spruce or pine, 4×6 ins.
Hub-plank, spruce or pine, 2×10 ins.

Dead-load:	Wood	250
	Steel	200
	Total	450 lbs. per lin.ft. of bridge.

Live-load for trusses, 75 lbs. per sq.ft. of roadway, =1,200 lbs. per lin.ft. of bridge.
Live-load for stringers and floorbeams, 100 lbs. per sq.ft.
Impact $=\dfrac{\text{range}^2}{2\text{ max.}}$, as given in Art. 17, Chap. V.
Horizontal force for proportioning laterals, 450 lbs. per lin.ft.
Unit-stresses, as given in Art. 18, Chap. V.
Rivets, $\frac{3}{4}$ in. diameter.

Stringers. The span of the stringers is 12 ft. 6 ins., and they are assumed to be 2 ft. apart c. to c. The dead-load on one stringer consists of its own weight as well as that of the floor planking supported thereby. Then, if the size of the stringer be assumed 3×12 ins., the dead-load may be computed as follows:

Stringer,	3×12 ins., 12 ft. 6 ins. long	=37.5
Floor planking,	3×24 ins., 12 ft. 6 ins. long	=75.0
		112.5 ft. b.m.

which, at 3 lbs. per ft. b.m., =337 lbs.

The live-load on one stringer $=2\times12.5\times100=2{,}500$ lbs. Then,

$$\text{Moments: Dead-load} = \frac{337\times12.5}{8} = 530$$

$$\text{Live-load:} = \frac{2{,}500\times12.5}{8} = 3{,}910$$

$$\text{Impact} = \frac{3{,}910^2}{2(530+3{,}910)} = \underline{1{,}720}$$

$$6{,}160 \text{ ft.-lbs.} = 73{,}920 \text{ in.-lbs.}$$

The permissible unit-stress for bending $=1{,}200$ lbs. per sq.in.; thus the section modulus S required $=\frac{M}{f}=\frac{73{,}920}{1{,}200}=61.6$; and therefore the assumed size of stringer (3×12 ins.) is suitable, for its $S=\frac{bh^2}{6}=\frac{3\times12^2}{6}=72$.

Dead-Load for Bridge. The weight of timber per lineal foot of bridge will now be computed, as follows:

Floor planking,	3 ins. $\times$16 ft.	=48
Nine stringers	3×12 ins.	=27
Two wheel-guards,	4×6 ins.	= 4
Two hub planks,	2×10 ins.	= 4
		83 ft. b.m. at 3 lbs. =249 lbs.

For round numbers, this timber construction is assumed to weigh 250 lbs. per lin.ft.

The approximate weight of steel per lineal foot of bridge will be determined by the formula given in Art. 17, Chap. V, in which $L=$ length of span in feet; thus,

$$\text{Weight of steel} = 2\tfrac{1}{2}L + 75 = (2\tfrac{1}{2} \times 50) + 75 = 200 \text{ lbs. per lin.ft.}$$

Floorbeams. The span used in computing the moments is the distance c. to c. of trusses, $=17$ ft. The dead-load consists of the weight of the beam itself, assumed to be 42 lbs. per ft. evenly distributed, plus the weight of the timber floor, which covers the central 16 ft. only, and is equal to $250 \times 12.5 = 3{,}100$ lbs. The live-load is also distributed over the central 16 ft., and is equal to $16 \times 12.5 \times 100 = 20{,}000$ lbs.

$$\begin{array}{llrr}
\text{End shear: Dead-load} & = 42 \times \dfrac{17}{2} = & 350 & \\
& + \dfrac{3{,}100}{2} = & \underline{1{,}550} & 1{,}900 \\
\text{Live-load} & = \dfrac{20{,}000}{2} = & & 10{,}000 \\
\text{Impact} & = \dfrac{10{,}000^2}{2(1{,}900+10{,}000)} = & & \underline{4{,}200} \\
& & & 16{,}100 \text{ lbs.}
\end{array}$$

The moment due to the weight of the beam is obtained by the formula, $M = \frac{wl^2}{8}$; but that due to the weight of the floor, as well as the live-load, is equal to the end reaction thereof, multiplied by its distance from the centre of the span, minus the portion of the load on one side of the centre multiplied by the distance of its centre of gravity from the centre of the span; thus,

$$\begin{array}{llrr}
\text{Moments: Dead-load} & = \dfrac{42 \times 17^2}{8} & = 1{,}500 & \\
& + 1{,}550 \times (8.5-4) & = \underline{6{,}975} & 8{,}475 \\
\text{Live-load} & = 10{,}000 \times (8.5-4) = & & 45{,}000 \\
\text{Impact} & = \dfrac{45{,}000^2}{2(8{,}475+45{,}000)} = & & \underline{18{,}925} \\
& & & 72{,}400 \text{ ft.-lbs.}
\end{array}$$

Then, $72{,}400 \times 12 = 868{,}800$ in.-lbs.: and $S = \dfrac{M}{f} = \dfrac{868{,}800}{16{,}000} = 54.3$.
A 15-inch I-beam at 42 lbs. will be used. Its $S = 58.9$.

Dead-Load Stresses in Trusses. In determining the stresses in the various members of the trusses, the total dead-load of 450 lbs. per lin.ft. of bridge is assumed to be concentrated at the lower panel-points. The panel dead-load for one truss is equal to one-half of the total dead-load per linear foot, multiplied by the length of one panel. At each of the panel-points *c*, *e*, and *g*, there is a full panel load; while at panel-points *a* and *i* the concentrations are each equal to one-half of a panel load; but these latter concentrations, which are supported directly by the masonry, do not affect the stresses in the trusses, and may thus be neglected.

Since the truss and its loads are symmetrical, the reaction at either end is equal to one-half of the total load—or one and one-half panel loads.

The shear in any panel is equal to the end reaction, minus any loads between this end and the panel considered; and the stress in any web-member situated in this panel is equal to the shear, multiplied by the length of the member, and divided by the depth of the truss.

The shear in panel *ac* is equal to the reaction at *a*; and the stresses in members *aB* and *Bc*, which lie in this panel, are equal to one another but are of opposite kind. Thus the stress in *aB* is compression, while that in *Bc* is tension.

The shear in panel *ce* is equal to the reaction at *a*, minus the load at *c*; and the stresses in members *cD* and *De* are also equal to one another but of opposite kind.

The bending moment at any panel-point of the top or bottom chord is equal to the algebraic sum of the moments, about this point, of the external vertical forces on either side thereof; and the stress in the opposite chord section is equal to this bending moment, divided by the depth of the truss.

For the stress in the bottom chord member *ac*, moments are taken about panel-point *B*. Now the only external force on the left of this point is the reaction at *a*; thus the bending moment at *B* is equal to the reaction at *a*, multiplied by its horizontal distance from *B*.

For the stress in the top chord member *BD*, moments are taken about panel-point *c*. The only external force on the left of this point is the reaction at *a*; thus the bending moment at *c* is equal to the reaction at *a*, multiplied by its horizontal distance from *c*.

For the stress in the bottom chord member *ce*, moments are taken about panel-point *D*. The external forces on the left of this point

are the reaction at *a* and the load at *c*; thus the bending moment at *D* is equal to the reaction at *a*, multiplied by its horizontal distance from *D*, minus the load at *c*, multiplied by its horizontal distance from *D*.

For the stress in the top chord member *DF*, moments are taken about panel-point *e*. The external forces on the left of this point are the reaction at *a* and the load at *c*; thus the bending moment at *e* is equal to the reaction at *a*, multiplied by its horizontal distance from *e*, minus the load at *c*, multiplied by its horizontal distance from *e*.

With the above explanation, the following table of stresses should be readily understood:

TABLE OF DEAD-LOAD STRESSES

Panel dead-load for one truss		$=\frac{450}{2}\times 12.5$	=	2,800
Reaction at *a*		$= 2{,}800\times 1\frac{1}{2}$	=	4,200
Shear in panel	ac	$= 4{,}200-0$	=	4,200
" "	ce	$= 4{,}200-2{,}800$	=	1,400
Moment at panel-point	B	$= 4{,}200\times 6.25$	=	26,250
" "	c	$= 4{,}200\times 12.5$	=	52,500
" "	D	$=(4{,}200\times 18.75)-(2{,}800\times 6.25)$	=	61,250
" "	e	$=(4{,}200\times 25)-(2{,}800\times 12.5)$	=	70,000
Stress in diagonal	aB	$= 4{,}200\times\frac{8.67}{6}$	=	+ 6,100
" "	Bc	$= 4{,}200\times\frac{8.67}{6}$	=	− 6,100
" "	cD	$= 1{,}400\times\frac{8.67}{6}$	=	+ 2,000
" "	De	$= 1{,}400\times\frac{8.67}{6}$	=	− 2,000
Stress in chord section	ac	$=26{,}250\div 6$	=	− 4,400
" "	BD	$=52{,}500\div 6$	=	+ 8,750
" "	ce	$=61{,}250\div 6$	=	−10,200
" "	DF	$=70{,}000\div 6$	=	+11,700

In the vertical members at *c*, *e*, and *g*, which serve to stiffen the top chord, there are no direct or calculable stresses.

Live-Load Stresses in Trusses. The live-load is concentrated at the panel-points of the bottom chord by means of the stringers and

floorbeams; the panel live-load for one truss being equal to one-half of the total live-load per linear foot multiplied by the length of a panel.

The maximum shear in panel *ac* occurs when the live-load covers the span; and, consequently, this condition of loading produces the maximum compression in member *aB*, as well as the maximum tension in member *Bc*.

Table of Live-load Stresses

Panel live-load for one truss	$\frac{1,200}{2} \times 12.5$	=	7,500
Reaction at *a* (loads at *c*, *e*, and *g*)	$= 7,500 \times 1\frac{1}{2}$	=	11,250
" (loads at *e* and *g*)	$= 7,500 \times \frac{3}{4}$	=	5,625
" (load at *g* only)	$= 7,500 \times \frac{1}{4}$	=	1,875
Moment at panel-point *B*	$= 11,250 \times 6.25$	=	69,500
" " *c*	$= 11,250 \times 12.5$	=	139,000
" " *D*	$= (11,250 \times 18.75) - (7,500 \times 6.25)$	=	161,700
" " *e*	$= (11,250 \times 25) - (7,500 \times 12.5)$	=	184,400
Stress in diagonal *aB*	$= 11,250 \times \frac{8.67}{6}$	=	+ 16,250
" " *Bc*	$= 11,250 \times \frac{8.67}{6}$	=	− 16,250
" " *cD*	$= 5,625 \times \frac{8.67}{6}$	=	+ 8,100
" " *De*	$= 5,625 \times \frac{8.67}{6}$	=	− 8,100
" " *eF*	$= 1,875 \times \frac{8.67}{6}$	=	+ 2,700
" " *Fg*	$= 1,875 \times \frac{8.67}{6}$	=	− 2,700
Stress in chord section *ac*	$= 69,500 \div 6$	=	− 11,600
" " *BD*	$= 139,000 \div 6$	=	+ 23,200
" " *ce*	$= 161,700 \div 6$	=	− 26,900
" " *DF*	$= 184,400 \div 6$	=	+ 30,700

The maximum positive shear in panel *ce* occurs when the live-load extends from the right-hand abutment to a point between the panel-points *c* and *e*; for the shear in this panel is equal to the reaction at *a*, minus the load at *c*. By advancing the load towards the panel-point *c*, the reaction at *a* is increased, but also the concentration at this panel-point. Now the exact position of the load for producing the maximum shear may be found by trial, or by the calculus; but the customary conventional method of dealing with a uniform live-load is to assume that it covers the panel in which the shear is required,

and to neglect the half-panel concentration at the adjacent left-hand panel-point. Therefore, for the maximum shear in this panel, it will be assumed that there are full panel loads concentrated at e and g, with no load at c. For this condition of loading, the reaction at a is equal to one-half of the load at e, plus one-quarter of the load at g; and the maximum shear in the panel is equal to the reaction. This condition gives the maximum compression in member cD, and the maximum tension in member De.

The maximum positive shear in panel eg would occur with a full panel load concentrated at g, and no loads at c and e. For this case, the reaction at a, and consequently the shear in panel eg, is equal to one-quarter of the load at g. This condition gives the maximum compression in member eF, and the maximum tension in member Fg.

The maximum moment at any point occurs when the live-load covers the span; thus the live-load stresses in the chords are determined in exactly the same manner as those due to the dead-load.

Impact Stresses in Trusses. The effect on a structure of a moving or live-load is undoubtedly greater than that of a fixed or dead-load. This additional stress, which is called impact, has not yet been determined satisfactorily, but is here provided for by the formula,

$$\text{Impact} = \frac{\text{range}^2}{2\ \text{max.}}.$$

In this formula *range* represents the total calculated stress in a member produced by the live-load; or, if the live-load tend to cause in a member alternate stresses of tension and compression, then the *range* is the numerical sum of these alternate live-load stresses. Again, *max.* represents the maximum stress in a member due to the dead- and live-loads (computed statically) which can occur at any one time.

In member aB the range of live-load stress $=16{,}250$ lbs.; and the maximum stress occurring at any one time is the sum of the dead- and live-load stresses, $=6{,}100+16{,}250=22{,}350$ lbs. Then

$$\text{Impact} = \frac{16{,}250^2}{2\times 22{,}350} = 5{,}900 \text{ lbs.},$$

as shown in Fig. 75.

In member Bc, the dead- and live-load stresses are the same

numerically as those in aB, although of opposite sign; and thus the impact is also the same.

In member cD, which corresponds to member Fg, the live-load stresses are +8,100 lbs. and −2,700 lbs., the numerical sum of which =10,800 lbs., being the range; and the maximum stress occurring at any one time is the sum of the dead-load stress (+2,000 lbs.) and the live-load stress (+8,100 lbs.) = 10,100 lbs. Then

$$\text{Impact} = \frac{10{,}800^2}{2 \times 10{,}100} = 5{,}800 \text{ lbs.}$$

In member De, the dead-load, live-load, and impact stresses are the same numerically as those in cD, but of opposite sign.

In member ac, the range = 16,600 lbs.; and the maximum = 4,400 + 16,600 = 21,000 lbs. Then

$$\text{Impact} = \frac{16{,}600^2}{2 \times 21{,}000} = 6{,}500 \text{ lbs.}$$

In member BD, the range = 23,200 lbs.; and the maximum = 8,750 + 23,200 = 31,950 lbs. Then

$$\text{Impact} = \frac{23{,}200^2}{2 \times 31{,}950} = 8{,}400 \text{ lbs.}$$

In member ce, the range = 26,900 lbs.; and the maximum = 10,200 + 26,900 = 37,100 lbs. Then

$$\text{Impact} = \frac{26{,}900^2}{2 \times 37{,}100} = 9{,}800 \text{ lbs.}$$

In member DF, the range = 30,700 lbs.; and the maximum = 11,700 + 30,700 = 42,400 lbs. Then

$$\text{Impact} = \frac{30{,}700^2}{2 \times 42{,}400} = 11{,}100 \text{ lbs.}$$

Proportioning of Truss Members. The dead-load, live-load, and impact stresses are summarized in Fig. 75.

For the tension members, the required sectional areas are obtained by dividing the total stresses by the permissible unit-stress of 16,000 lbs. per sq.in.; and the net sectional areas of the angles used are determined by allowing for two $\frac{7}{8}$-inch holes in each angle, the intention being to use $\frac{3}{4}$-inch rivets.

For the compression members, the permissible unit-stress for each individual case is obtained from Table I, Art. 15; and the sections are proportioned so that the unsupported length of no member exceeds 120 times its least radius of gyration.

For member cD, the total stress $= +15{,}900$ lbs., and its length $= 104$ ins. Then, assuming two angles $3 \times 2\frac{1}{2} \times \frac{5}{16}$ ins., with the 3-inch legs back to back but separated $\frac{1}{2}$ in. on account of the connection plates, the least radius of gyration is found to be 0.94 in. Thus the ratio $\frac{l}{r} = \frac{104}{0.94} = 110$, which, by the column table, corresponds to a permissible unit-stress of 7,180 lbs. per sq.in.; and $15{,}900 \div 7{,}180 = 2.22$ sq.ins. required. The sectional area of the assumed angles is greater than this, but the general practice at present prohibits metal in highway bridges under $\frac{5}{16}$ in. thick; and, if smaller angles were used, the ratio $\frac{l}{r}$ would exceed 120.

For member De, the principal stress $= -15{,}900$ lbs., requiring one sq.in. of net sectional area. But this member may at times be subject to a small amount of compression; and thus two angles $3 \times 2\frac{1}{2} \times \frac{5}{16}$ ins. are also used here.

The top chord is supported vertically at intervals of 6 ft. 3 ins.; but is held laterally only by the vertical posts, which are 12 ft. 6 ins. apart and knee-braced to the floorbeams; consequently, for equal strength in both directions, the radius of gyration about the vertical axis of this member should be twice as great as that about the horizontal axis.

Assuming for member BD, two angles $4 \times 3 \times \frac{3}{8}$ ins., with the 3-inch legs back to back but separated $\frac{1}{2}$ in., the radius of gyration about the vertical axis is found to be 1.98 ins., and that about the horizontal axis, 0.88 in.; consequently, the half-panel length (75 ins.) divided by this latter radius (0.88 in.), gives the greater value for $\frac{l}{r}$, and therefore determines the permissible unit-stress. Thus $\frac{l}{r} = \frac{75}{0.88} = 85$, which, by the column table, corresponds to 8,560 lbs. per sq.in.; and the total stress in the member, divided by this unit-stress, is the area required, viz.: $40{,}350 \div 8{,}560 = 4.72$ sq.ins., as shown; whereas the area of the assumed angles $= 4.96$ sq.ins.

For member DF the total stress $= +53{,}500$ lbs.; and the unsupported length, 75 ins., as before. Then, assuming two angles $4 \times 3 \times \frac{1}{2}$

ins., the least radius of gyration is found to be 0.86 in.; $\frac{l}{r}=\frac{75}{0.86}=87$, corresponding to a permissible unit-stress of 8,460 lbs. per sq.in.; and $53,500 \div 8,460 = 6.33$ sq.ins. required, whereas the area provided is 6.50 sq.ins.

For member *aB* the total stress $= +28,250$ lbs., and the length $=104$ ins. In order to continue the outline of the top chord, which is desirable for the sake of appearance, two angles $4 \times 3 \times \frac{5}{16}$ ins. are used here, with the 3-inch legs back to back as before. The least radius of gyration is found to be 0.89 in.; then $\frac{l}{r}=\frac{104}{0.89}=117$, corresponding to a permissible unit-stress of 6,820 lbs. per sq.in.; and $28,250 \div 6,820 = 4.14$ sq.ins. required; whereas the area provided is 4.18 sq.ins.

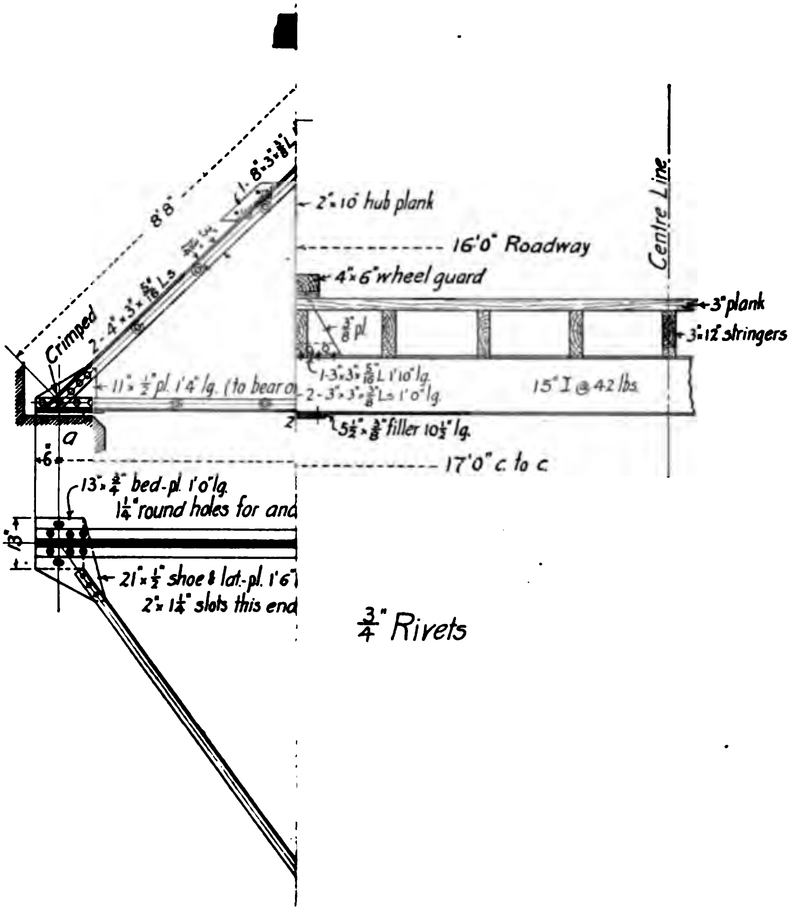
For the vertical posts, two angles $3 \times 3 \times \frac{5}{16}$ ins. are used.

Laterals. The lateral system, as shown in Fig. 75, is a horizontal truss of 50 ft. span and 17 ft. deep. There are four panels of 12 ft. 6 ins. each; and the length of the diagonals $=\sqrt{12.5^2+17^2}=21.1$ ft. The horizontal wind pressure is taken at 450 lbs. per lin.ft. of bridge; and thus a panel load $=450 \times 12.5 = 5,625$ lbs. As in the vertical trusses, there is a panel load concentrated at each point *c*, *e*, and *g*. The shear in the end panels is equal to one and one-half panel loads, $=5,625 \times 1\frac{1}{2} = 8,440$ lbs.; and the stress in the end diagonals $=8,440 \times \frac{21.1}{17} = 10,475$ lbs., which, at 16,000 lbs. per sq. in., $=0.65$ sq.in. required. One angle $2\frac{1}{2} \times 2\frac{1}{2} \times \frac{5}{16}$ ins. is used for each diagonal. Its net area, allowing for one $\frac{7}{8}$-inch hole, $=1.20$ sq.ins.

Details. Fig. 76 is a detail drawing showing an inside view of one-half of a truss, one-half cross-section and one-quarter plan of laterals. The trusses are supposed to be shipped rivetted up; thus all rivets, except those in the end connections of the floorbeams and laterals, are shop-driven.

The rivets throughout are $\frac{3}{4}$ in. diameter; then, by Table V, Art. 19, the value of one shop-driven rivet in single shear $=5,300$ lbs.; in bearing on $\frac{3}{8}$-inch plate $=6,760$ lbs.; and in bearing on $\frac{1}{2}$-inch plate $=9,000$ lbs. By Table VI, the value of one field-driven rivet in single shear $=3,975$ lbs.

The gusset-plates in the truss are $\frac{1}{2}$ in. thick throughout, the object in using this comparativelv thick metal being to obtain high bearing values for the rivets therein in order that the number required



[...]6. 50-Ft. Through Warren Girder. Detail Drawing.

will not be excessive. The sizes of the gusset-plates are determined by the number of rivets required in the connections, with a minimum pitch of 3 ins.; and the number of rivets required in any connection is obtained by dividing the stress in the member by the permissible bearing value of one rivet on the $\frac{1}{2}$-in. gusset-plates, this value being less than that of double shear. In accordance with what is generally accepted as good practice, no fewer than three rivets are employed in any connection.

Detail at *a*. The number of rivets required in the connection of the end post $= 28,250 \div 9,000 = 3.1$; and the number required in the bottom chord connection $= 27,500 \div 9,000 = 3$; whereas the drawing shows four in both cases. The angles of the end post are crimped to fit over those of the bottom chord. The gusset-plate is required to bear evenly on the shoe-plate, and thus transmit the vertical component of the stress in the end post without calling upon the rivets in the bottom chord angles for this purpose. The total reaction on the shoe- and bed-plates is equal to the vertical component of the stress in the end post, $= 28,250 \times \frac{6}{8.67} = 19,600$ lbs.; and, taking the permissible bearing on the abutment at 400 lbs. per sq.in., the area required in the bed-plate $= 19,600 \div 400 = 49$ sq.ins. The area of the plate used $= 12 \times 13 = 156$ sq.ins.; but practical considerations do not admit of employing a bed-plate of smaller dimensions. The shoe-plate, which is $\frac{1}{2}$ in. thick, is made larger than the bed-plate, and cut so as to serve as a lateral connection. Round holes $1\frac{1}{4}$ ins. diameter are provided in the shoe- and bed-plates at one end of the span for $1\frac{1}{8}$-inch anchor bolts; whereas at the other end, in order to allow for expansion, the shoe-plates are provided with $2 \times 1\frac{1}{4}$ in. slots.

Detail at *B*. Here, the connections of members *aB* and *Bc* each require 3.1 rivets, whereas four rivets are provided. If more than four rivets were required for member *Bc*, it would be better to provide lock-angles containing at least two rivets. But the stress in this member is small, and the area greatly exceeds that required; for two holes were allowed for in each angle, while there is actually only one; thus the single line of rivets as shown is entirely satisfactory. In the connection of member *BD*, the number of rivets required $= 40,350 \div 9,000 = 4.5$; and six rivets are provided. The bent plate which covers the hip is to give lateral stiffness, and cannot be depended upon to transmit any portion of the direct stresses.

Detail at *c*. The bottom chord is spliced $3\frac{1}{2}$ ins. to the left of the panel-point, partly by the $\frac{1}{2}$-inch gusset-plate and partly by the $\frac{5}{16}$-inch lateral plate. Now the net area at the point where the angles are cut should not be less than that required for member *ac*, viz., 1.72 sq.ins.; but it is evident that the whole width of the gusset-plate cannot be considered to resist the stress, as all the pull is on one edge, and, if the plate were to begin to fail at that edge, it would soon be torn across; and the same argument applies to the lateral plate. For the vertical gusset-plate, it is only safe to figure on a width equal to twice the rivet gauge in the angle, $=3\frac{1}{2}$ ins.; and from this should be deducted the diameter of the rivet hole, $\frac{7}{8}$ in. For the lateral plate, the effective width may be considered to be equal to the distance out-to-out of chord angles, $=6\frac{1}{2}$ ins., minus two $\frac{7}{8}$-inch rivet holes. Thus the net effective sectional area of the $\frac{1}{2}$-inch gusset-plate $=(3\frac{1}{2}-\frac{7}{8})\times\frac{1}{2}$ ins. $=1.31$ sq.ins.; while that of the $\frac{5}{16}$-inch lateral plate $=(6\frac{1}{2}-1\frac{3}{4})\times\frac{5}{16}$ ins. $=1.48$ sq.ins.; the total being 2.79 sq.ins. The value of the rivets at the left-hand end of the splice should be equal to or greater than the stress in *ac*, 27,500 lbs.; whereas the value of the rivets shown is as follows:

Three rivets bearing on $\frac{1}{2}$-inch gusset-plate, at 9,000	=27,000
Four rivets in lateral plate (single shear), at 5,300	=21,200
Total value of rivets in left-hand end of splice	=48,200 lbs.

Since this value is greater than the stress in member *ce*, it is unnecessary to compute the strength of the rivets in the right-hand end of the splice.

Detail at *D*. The top chord is spliced $1\frac{1}{2}$ ins. to the left of the panel-point, partly by the $\frac{1}{2}$-inch gusset-plate and partly by the $8\frac{1}{2}\times\frac{3}{8}$ in. cover-plate. The value of the rivets in the left-hand end of the splice is as follows:

Three rivets bearing on $\frac{1}{2}$-inch gusset-plate, at 9,000	=27,000
Six rivets in cover-plate (single shear), at 5,300	=31,800
Total value of rivets in left-hand end of splice	=58,800 lbs.

This value is not only greater than the stress in member *BD*, but also exceeds that in member *DF*.

Floorbeam Connection. The total end shear, as previously determined, $=16,100$ lbs.; and this force must be resisted by the rivets connecting the end angles to the web of the floorbeam. Now the rivets are located $2\frac{5}{8}$ ins. from the centre line of the truss, as shown in Fig. 76; and thus they are subject to a moment equal to 16,100 lbs. $\times 2\frac{5}{8}$ ins. $=42,260$ in.-lbs. This moment induces horizontal forces acting on the rivets, tending to push those above the centre of gravity towards the centre of the floorbeam, and to pull those below the centre of gravity in the opposite direction. The maximum horizontal force is applied to the outer rivets, and may be determined by dividing the bending moment by the section modulus of the rivets, which latter is computed as follows:

The vertical spacing of these rivets is 3 ins.; thus their several distances from the centre of gravity are $1\frac{1}{2}$ and $4\frac{1}{2}$ ins.; and their moment of inertia, taking one rivet as the unit of value, is equal to the sum of the products of all the rivets multiplied by the squares of their several distances from the neutral axis, $=2(1.5^2+4.5^2)=45$. The section modulus of the rivets is equal to this moment of inertia divided by the distance from the neutral axis to the outer rivets, $=\frac{45}{4.5}=10$.

Then the horizontal force acting both on the upper and on the lower rivet $=42,260\div 10=4,226$ lbs.

In addition to this horizontal force, there is a vertical load equal to the end shear, which is assumed to be equally distributed among the four rivets; thus the vertical load on each rivet $=16,100\div 4=4,025$ lbs.

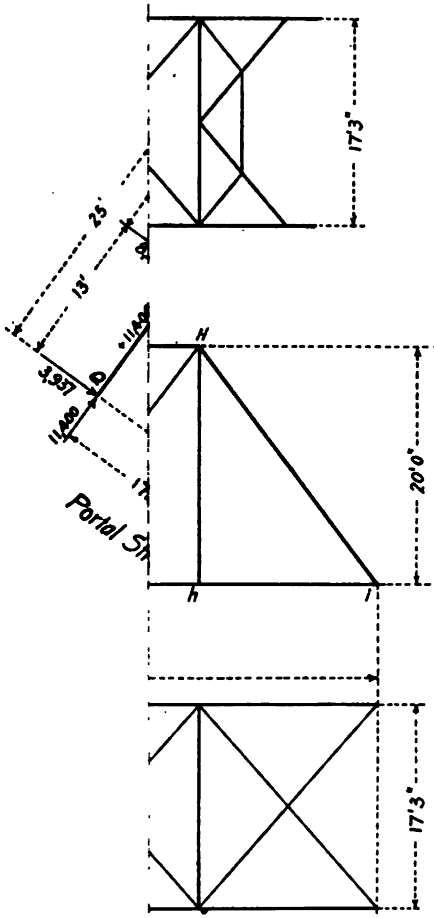
Therefore, the total force acting both on the upper and on the lower rivet is equal to the resultant of the horizontal and vertical forces determined above; and may be represented by the hypothenuse of a right-angled triangle, in which the horizontal side represents the force due to bending, and the vertical side, the direct load. Then, the total force acting on the outer rivets $=\sqrt{4,226^2+4,025^2}=5,830$ lbs.; and, since this is less than the permissible bearing value of a $\frac{3}{4}$-inch rivet on the $\frac{3}{8}$-inch web of the floorbeam, the connection is amply strong.

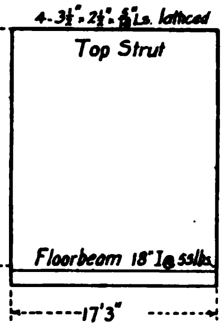
Connecting the end angles to the truss, there are six $\frac{3}{4}$-inch field-driven rivets in single shear, which, at 3,975 lbs. per rivet $=23,850$ lbs.

The top flange of the floorbeam is connected to the vertical post of the truss by a $\frac{3}{8}$-inch gusset-plate. The purpose of this connection is to give lateral support to the top chord.

Camber. Bridge trusses are constructed with a slight arch called camber. This adds nothing to their strength, but is provided principally to offset the deflection due to the dead- and live-loads. The camber is obtained by making the top chord slightly longer than the bottom chord; the usual rule is to increase the length of the top chord about $\frac{1}{8}$ in. for every 10 ft.

Estimated Weight. The weight of steel in this structure, as determined by a careful estimate made from the detail drawing, Fig. 76, is about 10,000 lbs. Thus the weight of metal per linear foot $= 10{,}000 \div 50 = 200$ lbs., which is just equal to that assumed for calculating the dead-load stresses. This may be considered as a coincidence; for it is usually impracticable to determine the weight of a bridge exactly before it has been designed. When the discrepancy between the assumed and actual weights is greater than about 10 per cent, it is advisable to refigure the bridge in accordance with the corrected weight.





Cross Section

Floorbeams.

Shear, dl 2,540
ll 12,000
imp 4,950
19,490 lbs.

Moment, dl 11,600
ll 55,500
imp 22,900
90,000 ft-lbs.

$= \frac{12}{16,000} = 67.6$

18" I @ 55 lbs. S = 88.4

Stress Sheet.

CHAPTER XI

THE DESIGN OF A PIN-CONNECTED PRATT TRUSS HIGHWAY BRIDGE

THIS type of bridge has been used very extensively in the past for spans varying from 75 ft. to 200 ft. The principal advantage of pin-connected spans over rivetted structures is in their comparatively easy and economical erection; but the latter are less subject to vibration and are generally longer lived.

The structure considered in this chapter will be designed in accordance with the following data:

Length, c. to c. of bearings, 120 ft. = 8 panels of 15 ft.;
Depth, c. to c. of chords, 20 ft.
Length of diagonal web-members $= \sqrt{20^2 + 15^2} = 25$ ft.
Width, c. to c. of trusses, 17 ft. 3 ins.
Roadway, 16 ft. clear.
Headroom, 14 ft. clear.
Floor planking, spruce or pine, 3 ins. thick.
Stringers, spruce or pine, 4×12 ins., about 2 ft. apart.
Wheel guards, spruce or pine, 4×6 ins.
Fence, spruce or pine, two lines $2\frac{1}{2} \times 6$ ins. on each truss.

Dead-load:	Wood	275
	Steel	375
	Total	650 lbs. per lin.ft. of bridge.

Live-load for trusses, 75 lbs. per sq.ft. of roadway, = 1,200 lbs. per lin.ft. of bridge.

Live-load for stringers, floorbeams, and hip verticals, 100 lbs. per sq.ft. of roadway.

$$\text{Impact} = \frac{\text{range}^2}{2\ \text{max.}},$$ as given in Art. 17, Chap. V.

Horizontal force for proportioning top laterals, 150 lbs. per lin.ft.
" " " " bottom laterals, 300 lbs. per lin.ft.
Unit-stresses as given in Art. 18, Chap. V.
Rivets, $\frac{3}{4}$ and $\frac{5}{8}$ in. diameter.

Stringers. The span of the stringers = 15 ft., and they are about 2 ft. apart, c. to c. The dead-load on one stringer consists of its own weight in addition to that of the floor planking supported thereby. Then, assuming the stringer to be 4×12 ins., the dead-load is as follows:

One stringer,	4×12 ins. 15 ft. long =	60
Floor planking,	3×24 ins. 15 ft. long =	90
		150 ft. B.M.,

which, at 3 lbs. per ft. B.M. = 450 lbs.

The live-load on one stringer $= 2 \times 15 \times 100 = 3{,}000$ lbs. Then

$$\text{Moment: Dead-load} = \frac{450 \times 15}{8} = 850$$

$$\text{Live-load} = \frac{3{,}000 \times 15}{8} = 5{,}625$$

$$\text{Impact} = \frac{5{,}625^2}{2(850 + 5{,}625)} = 2{,}450$$

$$8{,}925 \text{ ft.-lbs.} = 107{,}100 \text{ in.-lbs.}$$

The permissible unit-stress for bending = 1,200 lbs. per sq.in.; and thus the section modulus S required $= 107{,}100 \div 1{,}200 = 89.2$. The section modulus of the assumed stringer (4×12 ins.) $= \frac{4 \times 12^2}{6} = 96$.

Floorbeams. The effective span is the distance c. to c. of trusses = 17 ft. 3 ins. The dead-load consists of the weight of the beam itself, assumed at 55 lbs. per lin.ft., evenly distributed, plus the weight of the timber floor, $= 275$ lbs. $\times 15$ ft. $= 4{,}125$ lbs., distributed over the central 16 ft. The live-load $= 100$ lbs. $\times 16$ ft. $\times 15$ ft. $= 24{,}000$ lbs., which is also distributed over the central 16 ft. Then

End shear: Dead-load $= 55 \times \frac{17.25}{2} = 480$

$+\frac{4,125}{2} = \underline{2,060}$ $\quad 2,540$

Live-load $= \frac{24,000}{2}$ $\quad = 12,000$

Impact $= \frac{12,000^2}{2(2540+12,000)}$ $\quad = \underline{4,950}$

19,490 lbs.

Moment: Dead-load $= \frac{55 \times 17.25^2}{8}$ $\quad = 2,050$

$+2060(8.62-4) = \underline{9,550}$ $\quad 11,600$

Live-load $= 12,000(8.62-4)$ $\quad = 55,500$

Impact $= \frac{55,500^2}{2(11,600+55,500)}$ $\quad \underline{22,900}$

90,000 ft.-lbs.

Then $90,000 \times 12 = 1,080,000$ in.-lbs., which, divided by $16,000 = 67.6$ $= S$ required. An 18-inch I-beam at 55 lbs., for which $S = 88.4$, is used.

Dead-Load for Trusses. The weight of timber per linear foot of bridge may be computed as follows:

Floor planking,	3 ins. $\times$ 16 ft.	$= 48$
Nine stringers,	4×12 ins.	$= 36$
Two wheel guards,	4×6 ins.	$= 4$
Four fence rails,	$2\frac{1}{2} \times 6$ ins.	$= 5$
		93 ft. B.M. at 3 lbs. $= 279$ lbs.

The approximate weight of steel, by formula given in Art. 17, $= 2\frac{1}{2}L + 75 = (2.5 \times 120) + 75 = 375$ lbs. per lin.ft. of bridge.

Dead-Load Stresses in Trusses. As in the previous example, the total dead-load is assumed to be concentrated at the lower panel-points; the concentration at each panel-point of a single truss being equal to one-half of the total dead-load per linear foot of bridge, multiplied by the length of one panel.

The reaction at either end is equal to one-half of the panel concentrations; and the shear in any panel is equal to the reaction, minus any panel concentrations between this panel and the end support.

The stress is any diagonal is equal to the shear in the panel in which it is situated, multiplied by the length of the diagonal and divided by the depth of the truss. The stress in any post is equal to the vertical component of the stress in the diagonal which meets its upper end, and is therefore equal to the shear in the panel containing this diagonal. The hip verticals *Bb* and *Hh* are not posts, but tension members, and the stress therein is equal to the panel concentration applied at their lower extremity.

For the stress in *abc*, moments are taken about panel-point *B*. For the stress in *BC*, moments are taken about *c*; and, for the stress in *cd*, moments are taken about *C*; but *c* and *C* are the same distance horizontally from *a*, and thus the stresses in *BC* and *cd* are equal. For the stress in *CD* or *de*, moments are taken about *d* or *D*; and, for the stress in *DE*, moments are taken about *e*.

Live-Load Stresses in Trusses. The maximum positive shear in any panel, with reference to the left-hand support, occurs with a load at every panel-point to the right of the panel considered, with no loads to the left; and is thus equal to the left-hand reaction for this distribution of loads.

When the positive live-load shear in a panel exceeds the negative dead-load shear (as will be found in panel *ef*, with live-loads at *f*, *g*, and *h* only) counter-ties are required, because the main diagonals are incapable of resisting compression. In panel *fg*, with loads at *g* and *h*, as well as in panel *gh*, with load at *h* only, it will be found that the negative dead-load shear exceeds the positive live-load shear; therefore the diagonals in these panels will not be subject to compression; and, consequently, no counter-ties are required. But the effects of the maximum positive live-load shears in these panels will be computed in the following table of live-load stresses, as they comprise a part of the total range of stress used in figuring the impact. The maximum live-load stresses in the chords are determined in the same manner as the dead-load stresses.

TABLE OF LIVE-LOAD STRESSES

Panel live-load for one truss	$=\frac{1,200}{2}\times 15$	=	9,000
Reaction a (loads b to h)	$= 9,000\times\frac{28}{8}$	=	31,500
" (loads c to h)	$= 9,000\times\frac{21}{8}$	=	23,625
" (loads d to h)	$= 9,000\times\frac{15}{8}$	=	16,875
" (loads e to h)	$= 9,000\times\frac{10}{8}$	=	11,250
" (loads f to h)	$= 9,000\times\frac{6}{8}$	=	6,750
" (loads g and h)	$= 9,000\times\frac{3}{8}$	=	3,375
" (load h only)	$= 9,000\times\frac{1}{8}$	=	1,125
Moment at panel-point	$B=31,500\times 15$	=	472,500
" "	$c=(31,500\times 30)-(9,000\times 15)$	=	810,000
" "	$d=(31,500\times 45)-[9,000\times(15+30)]$	=	1,012,500
" "	$e=(31,500\times 60)-[9,000\times(15+30+45)]$	=	1,080,000
Stress in member	$aB=31,500\times\frac{25}{20}$	=+	39,375
" "	$Bc=23,625\times\frac{25}{20}$	=−	29,530
" "	$Cc=16,875\times 1$	=+	16,875
" "	$Cd=16,875\times\frac{25}{20}$	=−	21,090
" "	$Dd=11,250\times 1$	=+	11,250
" "	$De=11,250\times\frac{25}{20}$	=−	14,060
" "	$Ee=6,750\times 1$	=+	6,750
" "	$Ef=6,750\times\frac{25}{20}$	=−	8,440
" "	$Ff=0\times 1$	=	0
" "	$fG=3,375\times\frac{25}{20}$	=+	4,220
" "	$Gg=3,375\times 1$	=−	3,375
" "	$gH=1,125\times\frac{25}{20}$	=+	1,400
" "	$abc=472,500\div 20$	=−	23,625
" "	$BC=810,000\div 20$	=+	40,500
" "	$Cd=810,000\div 20$	=−	40,500
" "	$CD=1,012,000\div 20$	=+	50,625
" "	$de=1,012,000\div 20$	=−	50,625
" "	$DE=1,080,000\div 20$	=+	54,000
" "	$Bb=100\times\frac{16}{2}\times 15$	=−	12,000

TABLE OF DEAD-LOAD STRESSES

Panel dead-load for one truss	$=\frac{650}{2}\times 15$	$= 4,875$
Reaction at a	$= 4,875\times 3\frac{1}{2}$	$= 17,060$
Shear in panel ab	$=17,060-0$	$= 17,060$
" " bc	$=17,060-4,875$	$= 12,185$
" " cd	$=17,060-(4,875\times 2)$	$= 7,310$
" " de	$=17,060-(4,875\times 3)$	$= 2,435$
Moment at panel-point B	$=17,060\times 15$	$= 255,900$
" " c	$=(17,060\times 30)-(4,875\times 15)$	$= 438,675$
" " d	$=(17,060\times 45)-[4,875\times(15+30)]$	$= 548,325$
" " e	$=(17,060\times 60)-[4,875\times(15+30+45)]$	$= 584,850$
Stress in member aB	$=17,060\times\frac{25}{20}$	$=+\ 21,325$
" " Bc	$=12,185\times\frac{25}{20}$	$=-\ 15,230$
" " Cc	$= 7,310\times 1$	$=+\ 7,310$
" " Cd	$= 7,310\times\frac{25}{20}$	$=-\ 9,140$
" " Dd	$= 2,435\times 1$	$=+\ 2,435$
" " De	$= 2,435\times\frac{25}{20}$	$=-\ 3,040$
" " abc	$=255,900\div 20$	$=-\ 12,795$
" " BC	$=438,675\div 20$	$=+\ 21,930$
" " cd	$=438,675\div 20$	$=-\ 21,930$
" " CD	$=548,325\div 20$	$=+\ 27,420$
" " de	$=548,325\div 20$	$=-\ 27,420$
" " DE	$=584,850\div 20$	$=+\ 29,240$
" " Bb	$= 4,875\times 1$	$=-\ 4,875$

Impact Stresses in Trusses. The impact is computed by the formula

$$Impact=\frac{range^2}{2\ max.},$$

as for the 50-foot Warren girder. The impact stresses for the chords, end posts, and hip verticals, in which the *range* is represented by the maximum live-load stress, and the *max.*, by the dead-load plus the live-load stress, require no further explanation; but for the remaining members, some of which are subject to live-load stresses of opposite character, the appplication of the impact formula is more complicated. These members will be dealt with individually.

In member Bc, the dead-load stresses $= -15,230$ lbs. The live-

load stress in this member (with loads at panel-points c to h) $= -29,530$ lbs.; and, in the corresponding member gH at the opposite end of the span, with load at h only, the live-load stress $= +1,400$ lbs. Thus both Bc and gH may be subject to a live-load stress of $-29,530$ lbs. or $+1,400$ lbs.; and the numerical sum of these stresses constitutes the ***range***, which is equal to $29,530+1,400=30,930$ lbs. The maximum stress to which this member Bc is liable is the sum of the dead-load and the maximum live-load stresses $=15,230+29,530=44,760$ lbs. Then

$$Impact=\frac{\text{range}^2}{2\text{ max.}}=\frac{30,930^2}{2\times 44,760}=-10,640\text{ lbs.},$$

as shown in Fig. 77.

In member Cc, the dead-load stress $= +7,310$ lbs.; the live-load stress $= +16,875$ lbs.; and the live-load stress in the corresponding member $Gg = -3,375$ lbs. Thus the ***range*** $=16,875+3,375=20,250$ lbs.; the ***max.*** $=7,310+16,875=24,185$ lbs.; and the

$$Impact=\frac{20,250^2}{2\times 24,185}=+8,465\text{ lbs.}$$

In member Cd, the dead-load stress $= -9,140$ lbs.; the live-load stress $= -21,090$ lbs.; and the live-load stress in the corresponding member $fG = +4,220$ lbs. Thus the ***range*** $=21,090+4,220=25\ 310$ lbs.; the ***max.*** $=9,140+21,090=30,230$ lbs.; and the

$$Impact=\frac{25,310^2}{2\times 30,230}=-10,580\text{ lbs.}$$

In member Dd, the dead-load stress $= +2,435$ lbs., and the live-load stress $= +11,250$ lbs. Owing to the counter-tie Ef, which comes into action with loads at panel-points f, g, and h, there is no stress under this distribution of the live-load in Ff, which member corresponds to Dd. Therefore, the ***range*** in either of these members is simply the maximum live-load stress, as above; the ***max.*** $=2,435+11,250=13,685$ lbs.; and the

$$Impact=\frac{11,250^2}{2\times 13,685}=+4,615\text{ lbs.}$$

In member *De*, the dead-load stress $= -3{,}040$ lbs., and the live-load stress $= -14{,}060$ lbs. This member, also, is unaffected by any reversal of shear in the panel, for its corresponding member *eF* is relieved of compression by the counter-tie *Ef*; thus the *range* in *De* or *eF* is the live-load stress, $-14{,}060$ lbs.; the *max.* $= 3{,}040 + 14{,}060 = 17{,}100$ lbs.; and the

$$Impact = \frac{14{,}060^2}{2 \times 17{,}100} = -5{,}800 \text{ lbs.}$$

In members *Ee* and *Ef*, there is no dead-load stress; but, with the maximum positive live-load shear in panel *ef*, which occurs when panel-points *f*, *g*, and *h* only are loaded, the main tie *eF* will be slackened up, or thrown out of action, and thus the negative dead-load shear in this panel must be resisted by members *Ee* and *Ef*, tending to induce tension in the former and compression in the latter, and thereby reducing the live-load stresses therein. The total range of stress in these members is the difference between the live-load stress and the effect produced by the dead-load; and the *max.* is equal to the *range*.

In *Ee*, the live-load stress $= +6{,}750$ lbs., and the effect of the dead-load $= -2{,}435$ lbs.; thus the *range* and the *max.* $= 6{,}750 - 2{,}435 = 4{,}315$ lbs.; and the

$$Impact = \frac{4{,}315^2}{2 \times 4{,}315} = +2{,}155 \text{ lbs.}$$

The total stress in the member is the algebraic sum of the dead-load, live-load, and impact stresses, as shown.

In *Ef*, the live-load stress $= -8{,}440$ lbs.; and the effect of the dead-load $= +3{,}040$ lbs., being the same numerically as the dead-load stress in the main ties *De* or *eF*; thus the *range* and the *max.* $= 8{,}440 - 3{,}040 = 5{,}400$ lbs.; and the

$$Impact = \frac{5{,}400^2}{2 \times 5{,}400} = -2{,}700 \text{ lbs.}$$

The total stress in the member, as in *Ee*, is the algebraic sum of the dead-load, live-load, and impact stresses.

Bottom Laterals. The bottom lateral system, as shown in Fig. 77, is a horizontal truss of 120-foot span and 17 ft. 3 ins. deep.

There are eight panels of 15 ft. each; and the length of diagonals $=\sqrt{15^2+17.25^2}=22.86$ ft. The diagonals are to be designed to resist a horizontal force of 300 lbs. per lin.ft. of bridge, which is assumed to be a moving load. The maximum shears in the various panels are computed in the same manner as those for the live-load in the case of the vertical trusses; and the stresses in the diagonals are equal to these shears, multiplied by the length of the diagonals and divided by the width of the bridge c. to c. of trusses.

Although the horizontal force, or wind-load, produces additional stresses in the chords and floor beams, it is unusual to consider such additional stresses unless they exceed 25 per cent of those due to dead-load, live-load, and impact; or, in other words, for the rare combination of maximum stresses due to dead-load, live-load, impact, and wind force, the permissible unit-stress may be increased 25 per cent.

Table of Bottom Lateral Stresses

$$\text{Panel-load}=300\times 15 \qquad = \quad 4{,}500$$

$$\text{Stress in 1st diagonal}=4500\times\frac{28}{8}\times\frac{22.86}{17.25}=-20{,}870$$

$$\text{〃 2d 〃}=4500\times\frac{21}{8}\times\frac{22.86}{17.25}=-15{,}650$$

$$\text{〃 3d 〃}=4500\times\frac{15}{8}\times\frac{22.86}{17.25}=-11{,}175$$

$$\text{〃 4th 〃}=4500\times\frac{10}{8}\times\frac{22.86}{17.25}=-\ 7{,}450$$

Top Laterals. The top lateral system consists of a horizontal truss of six 15-foot panels, supported laterally at the hips B and H by the portal strut, as shown in Fig. 77. The depth of this horizontal truss $=17.25$ ft., and the length of the diagonals $=22.86$ ft., as for the bottom laterals. The assumed wind-load is 150 lbs. per lin.ft.; and the panel-load $=150\times 15=2{,}250$ lbs. The shear in the end panels is equal to $2\frac{1}{2}$ panel loads, $=5{,}625$ lbs.; and the stress in the end diagonals $=5{,}625\times\frac{22.86}{17.25}=7{,}450$ lbs. Since this stress is so small, it is unnecessary to compute the stresses for the intermediate diagonals.

Portal Strut. Referring to Fig. 77, it is assumed that a horizontal force equal to $3\frac{1}{2}$ top lateral panel-loads, $=2,250\times3\frac{1}{2}=7,875$ lbs., is applied to the portal strut at the point M, as shown, and that the horizontal reactions at K and Q for this force are each equal to $7,875\times\frac{1}{2}=3,937$ lbs.

When the bridge is fully loaded, it may be further assumed that the posts are fixed in the plane of the portal strut at the points K and Q, and that the points of contraflexure are midway between these points and the points L and P. Thus the bending moment due to the wind-load, which may be used in conjunction with the total dead-load, live-load, and impact stresses, is maximum at the foot of the posts and at the lower connections of the portal strut; and is equal to the horizontal reaction at K or Q, multiplied by one-half the distance between these points and the points L or P, $=\frac{7,875}{2}\times\frac{13}{2}=25,600$ ft.-lbs. $=307,200$ in.-lbs.

But when the bridge is unloaded, the lower ends of the posts are not perfectly fixed; and, for the purpose of designing the portal strut, it is advisable to assume that these ends are hinged in the plane of the portal strut, as well as in the planes of the main trusses. In this case the maximum bending moment, which is at L or P, is equal to the horizontal reaction at K or Q, multiplied by the distance KL or QP, $=\frac{7,875}{2}\times13=51,200$ ft.-lbs. $=614,400$ in-.lbs. Now these moments must be balanced by induced forces at M and O, acting in the same direction as the horizontal reactions at K and Q; and the lever arm, about the points L and P, of these induced forces at M and O is equal to the distance ML or $OP=12$ ft. Therefore, the forces required at M and O to balance the moments about L and P which result from the horizontal reactions at K and $Q=\frac{7,875}{2}\times\frac{13}{12}=4,266$ lbs. These forces act in the same direction as the horizontal reactions at K and Q, and must be balanced by equal but opposite external forces acting in the direction of the applied wind force, as shown. At L and P, there are horizontal forces equal to the horizontal reaction at K or Q, plus the induced force at M or $O=3,937+4,266=8,203$ lbs. These forces act in the opposite direction to the forces which they balance, and must be resisted by equal forces acting in the opposite direction to the applied wind force, as shown. In addition to the horizontal reactions at K and Q, there are also reactions in the plane of the portal

strut, which are equal to the applied wind force at M, multiplied by the length of the post and divided by their distance c. to c., $=7,875 \times \frac{25}{17.25}=11,400$ lbs. That at K acts downwards, and that at Q upwards, as shown.

Having determined the external forces acting upon the portal strut, the stresses therein may readily be found graphically, or computed analytically. Thus the stress in MN is equal to the applied wind force at M, plus the force supplied to balance that induced through bending, $=7,875+4,266=+12,141$ lbs. The stress in NO is equal to the force supplied at O to balance that induced through bending $=-4,266$ lbs. The stress in PN is equal to the horizontal force supplied at P, multiplied by the length of the member and divided by one-half the width of the portal strut, $=8,203\times\frac{14.75}{8,625}=+14,000$ lbs. The stress in NL is of the same magnitude as that in PN, but of opposite character. There are no stresses in the short diagonals, or in the horizontal member between the main diagonals; but these members are supplied for stiffening purposes and general effect.

Proportioning of Truss Members and Lateral Systems. The total stresses in the various truss members, as well as in the laterals and portal strut, are shown in Fig. 77; also the sectional areas required and provided.

The sectional areas required for the tension members are determined by dividing the total stress therein by the permissible unit-stress of 16,000 lbs. per sq.in.; except that, for the counter-ties in panels *de* and *ef*, which are of iron, the permissible unit-stress is taken at 12,000 lbs. per sq.in.

For the compression members, the permissible unit-stress for each individual case is obtained from Table I, Art. 15; and the sections are proportioned so that the unsupported length of no member exceeds 120 times its least radius of gyration.

The top chords and end posts are composed of 9-inch channels, 10 ins. back to back, and latticed on both upper and lower sides. It should be noted that the radius of gyration varies in the different weights of these channels, which accounts for the differences in the permissible unit-stresses for the several panels.

The end posts are first designed without taking into account the stresses due to wind-load, and it is found that two 9-inch channels at 20 lbs. per ft. are required, their sectional area being 11.76 sq.ins.

But these members should also be investigated for two other conditions:

First. Considering the bending moment due to the wind-load (assuming the ends of the posts fixed) combined with the total stress due to dead-load, live-load, and impact; for which condition a maximum fibre-stress of 12,000+25 per cent=16,000 lbs. per sq.in. may be permitted.

Second. Considering the bending moment due to wind-load (assuming the ends of the posts hinged) combined with the direct stresses due to dead- and wind-loads only; and for this condition a maximum fibre stress of 16,000 lbs. per sq.in. may also be permitted.

For two 9-inch channels at 20 lbs., separated 10 ins. back to back, the moment of inertia about an axis through the centre of gravity and parallel to the webs, is found to be 371.4; the distance y_1 from this axis to the outer fibres=7.65 ins.; and the section modulus $S=\frac{I}{y_1}=\frac{371.4}{7.65}=48.55$.

For the first condition, the bending moment, which has been computed in connection with the portal strut, =307,200 in.-lbs.; and the total direct stress, as given in Fig. 77, =+73,450 lbs. Then

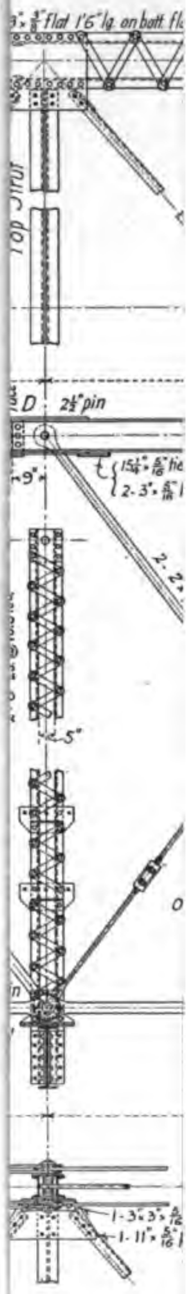
$$\begin{aligned} 73{,}450 \div 11.76 &= 6{,}250 \\ 307{,}200 \div 48.55 &= \underline{6{,}330} \\ & 12{,}580 \text{ lbs. per sq.in.} \end{aligned}$$

Thus the section is ample for this condition.

For the second condition, the bending moment=614,400 in.-lbs.; the dead-load stress=+21,325 lbs., and the direct wind-load stress =±11,400 lbs.; the total direct stress being equal to 21,325+11,400 =+32,725 lbs. Then

$$\begin{aligned} 32{,}725 \div 11.76 &= 2{,}780 \\ 614{,}400 \div 48.55 &= \underline{12{,}660} \\ & 15{,}440 \text{ lbs. per sq.in.} \end{aligned}$$

Therefore the assumed section composed of two 9-inch channels at 20 lbs. satisfies all conditions.


Top Strut
D
2½" pin
9"
5"

For the vertical posts, two 6-inch channels at 10.5 lbs. are used, latticed 5 ins. back to back. Their sectional area is considerably greater than required for the actual stresses; but, with smaller channels, the length would exceed 120 times the least radius of gyration; and the lighter weight of the 6-inch channels has a web of only 0.20 in., which is entirely too thin.

For the hip verticals, which are tension members, the same section used for the vertical posts is employed. The object in adopting stiff members at these points is to provide rigid connections for the floorbeams and laterals.

In the portal strut, the stresses are light; and thus the sizes of the principal members are determined by the condition that the ratio $\frac{l}{r}$ shall not exceed 120, and also that the minimum thickness of metal shall not be less than $\frac{5}{16}$ in. The members are each composed of two angles $3\frac{1}{2}\times 2\frac{1}{2}\times\frac{5}{16}$ in., latticed and placed as shown in the detail drawing, Fig. 78. The minor members are made of two angles $2\frac{1}{2}\times 2\frac{1}{2}\times\frac{5}{16}$ in. stayed by tie-plates only.

The sectional area of the top laterals is greatly in excess of that required for the stress. They are composed of two angles $2\frac{1}{2}\times 2\frac{1}{2}\times\frac{5}{16}$ in. latticed in vertical planes. The reason for making these members comparatively stiff is to lessen the vibrations to which they are subject.

The top struts are composed of four angles $3\frac{1}{2}\times 2\frac{1}{2}\times\frac{5}{16}$ in., with the $3\frac{1}{2}$ in. legs horizontal, and the $2\frac{1}{2}$-inch legs back to back but separated sufficiently to accommodate the latticing. The section of these members is determined by the condition that the ratio $\frac{l}{r}$ shall not exceed 120.

Owing to the very efficient top laterals and portal struts, it is thought unnecessary to provide vertical sway bracing at the intermediate posts.

Details. In Fig. 78, which is a general detail drawing of the bridge, is shown an inside elevation of one-half of truss, together with top and bottom laterals, one-half cross-section at centre of span, one-half of portal strut, also pier members. The truss and laterals are laid out to a smaller scale than that used for the details, the object being to keep the drawing within practicable limits and to make the essential details as clear as possible.

Pin Moments. One of the first things to consider in detailing a bridge of this type is the bending moments on the pins. Now the

pins in the bottom chords and those at the hips are usually found to be subject to their maximum bending moments when the bridge is fully loaded; but, on the pins at the intermediate panel-points of the top chords, the maximum moments occur with the greatest stresses in the diagonals connected thereby.

Before computing the pin moments, it is necessary first to determine, for the condition of loading considered, the stresses which occur simultaneously in the connected members. It is usually found to be most convenient to resolve the stresses in the diagonal members into their vertical and horizontal components, and to compute the vertical and horizontal bending moments on the pins separately. (It should be noted that the sum of the vertical forces as well as of the horizontal forces acting on a pin must always be zero.) Then the resultant moment at any point of a pin is equal to the hypothenuse of a right-angled triangle whose vertical and horizontal sides are equal respectively to the vertical and horizontal bending moments at that point.

Having found the maximum moment in inch-pounds, the size of pin required is found by inspection of Table VII herewith, wherein the bending values are obtained by multiplying the section modulus S of each size of pin by the permissible maximum fibre-stress of 24,000 lbs. per sq.in.

A pin may be large enough to resist the bending moment thereon, and yet too small for bearing; so care must be taken to ensure that the bearing of all members on the pin does not exceed the permissible limit of 24,000 lbs. per sq.in., as called for in Art. 18.

It is seldom necessary to consider the shear, for a pin which is large enough to meet the requirements for bending and bearing is usually of ample size to resist the maximum shear, which latter, however, should not exceed 12,000 lbs. per sq.in.

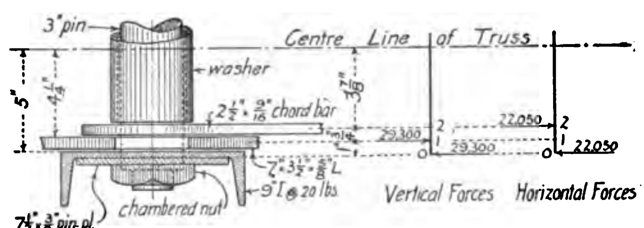
Since the pins are loaded symmetrically about their transverse centre line, it is only necessary to consider the forces acting on one side of this centre line.

Table VII

BENDING VALUES OF PINS WITH EXTREME FIBRE-STRESS OF 24,000 POUNDS PER SQUARE INCH

Diam. of Pin in Ins.	Area of Pin in Sq.ins.	Bending Value in In.-lbs.	Diam. of Pin in Ins.	Area of Pin in Sq.ins.	Bending Value in In.-lbs.	Diam. of Pin in Ins.	Area of Pin in Sq.ins.	Bending Value in In.-lbs.
1	0.785	2,350	3	7.069	63,600	5	19.635	294,500
1⅛	0.994	3,350	3⅛	7.670	71,900	5⅛	20.629	317,200
1¼	1.227	4,600	3¼	8.296	80,900	5¼	21.648	340,900
1⅜	1.485	6,100	3⅜	8.946	90,600	5⅜	22.691	365,900
1½	1.767	7,950	3½	9.621	101,000	5½	23.758	392,000
1⅝	2.074	10,100	3⅝	10.321	112,200	5⅝	24.850	419,400
1¾	2.405	12,600	3¾	11.045	124,200	5¾	25.967	448,000
1⅞	2.761	15,500	3⅞	11.793	137,000	5⅞	27.109	477,800
2	3.142	18,800	4	12.566	150,800	6	28.274	508,900
2⅛	3.547	22,500	4⅛	13.364	165,400	6⅛	29.465	541,400
2¼	3.976	26,900	4¼	14.186	180,800	6¼	30.680	575,300
2⅜	4.430	31,500	4⅜	15.033	197,300	6⅜	31.919	610,400
2½	4.909	36,800	4½	15.904	214,700	6½	33.183	647,000
2⅝	5.412	42,600	4⅝	16.800	233,200	6⅝	34.472	685,100
2¾	5.940	48,900	4¾	17.721	252,500	6¾	35.785	724,700
2⅞	6.492	56,000	4⅞	18.665	273,000	6⅞	37.122	765,600

Pin *a*. In Fig. 79 is shown one-half of pin *a* with the members connected thereto; also the vertical and horizontal components of the stresses in these members.



Pin and Connected Members at a.

Fig. 79.

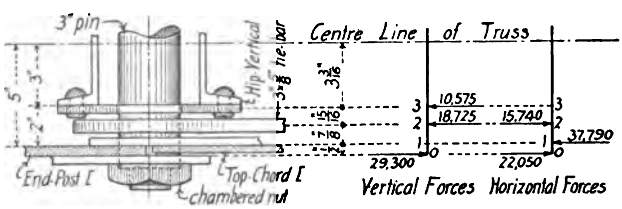
The vertical component of the stress in the end post is equal to the total stress in this member, multiplied by the depth of the truss and divided by the length of the member, $=73{,}450\times\frac{20}{25}=58{,}600$ lbs.;

and the reaction of the shoe standards is equal to this component; thus the downward force at either bearing of the post, and the upward force at either shoe standard are both equal to $58,600 \times \frac{1}{2} = 29,300$ lbs., as shown. The horizontal component of the stress in the end post is equal to the total stress in the bottom chord; thus the force acting towards the right at either chord bar and the force acting towards the left at either bearing of the end post are both equal to $44,100 \times \frac{1}{2} = 22,050$ lbs., as shown. In determining the distances between the various forces, about $\frac{1}{8}$ in. is allowed for clearance between any two adjacent members. The points of loading on the pin are marked with the numbers 0, 1, and 2, and the vertical and horizontal moments are computed in inch-pounds, as follows:

Vertical moment at	$(1) = 29,300 \times 1$	$= 29,300$
" "	$(2) = 29,300 \times (1\frac{3}{4} - \frac{3}{4})$	$= 29,300$
Horizontal moment at	$(1) = 22,050 \times 1$	$= 22,050$
" "	$(2) = 22,050 \times 1\frac{3}{4}$	$= 38,600$
Resultant moment at	$(2) = \sqrt{29,300^2 + 38,600^2}$	$= 48,600$

On referring to Table VII, it will be found that a $2\frac{3}{4}$-in. pin, which has a bending value of 48,900 in.-lbs., is the size actually required.

Pin *B*. In Fig. 80 is shown one-half of pin *B* with attached members. It should be noted that the pin plate for the top chord is



Pin and Connected Members at B.

FIG. 80.

on the inside, thus making the centre of bearing for this member coincide practically with the back of the channel; whereas the pin plate for the end post is on the outside, which makes the centre of bearing for this member coincide nearly with the outer edge of the

web; the two bearings being about $\frac{1}{2}$ in. c. to c., as shown. The tie bar is placed about midway between the top chord and the hip vertical; thus the rivet heads in these members require to be flattened only, instead of being countersunk.

With the bridge fully loaded the stress in the end post, as shown in Fig. 77, $=73,450$ lbs., the vertical and horizontal components of which are 58,600 lbs. and 44,100 lbs.; thus the upward force at either bearing of the end post $=58,600\times\frac{1}{2}=29,300$ lbs., and the force acting towards the right $=44,100\times\frac{1}{2}=22,050$ lbs. The stress in the hip vertical $=21,150$ lbs.; so the downward force at either bearing of this member $=21,150\times\frac{1}{2}=10,575$ lbs. The stress in the top chord $=75,580$ lbs.; and thus the force acting towards the left at either bearing $=75,580\times\frac{1}{2}=37,790$ lbs. The maximum stress in the tie bar, as given in Fig. 77, does not occur with the condition of loading assumed for maximum bending moment on the pin; but, since all of the other forces are known, it is only necessary to make the vertical and horizontal components of the stress in this member such that the sum of the vertical and of the horizontal forces acting on the pin shall be zero. Therefore the downward force at either tie bar $=29,300-10,575=18,725$ lbs.; and the force acting towards the right $=37,790-22,050=15,740$ lbs.

These vertical and horizontal forces, with the distances between them, are shown in Fig. 80. Then

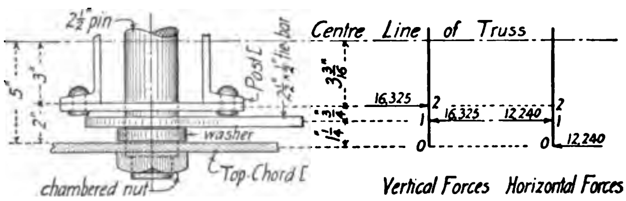
Vertical moment at	$(1)=29,300\times\frac{1}{2}$	$=14,650$
" "	$(2)=29,300\times(\frac{1}{2}+\frac{7}{8})$	$=40,300$
" "	$(3)=[29,300\times(\frac{1}{2}+\frac{7}{8}+\frac{15}{16})]-[18,725\times\frac{15}{16}]$	$=50,300$
Horizontal moment at	$(1)=22,050\times\frac{1}{2}$	$=11,025$
" "	$(2)=[22,050\times(\frac{1}{2}+\frac{7}{8})]-[37,790\times\frac{7}{8}]$	$=2,800$
" "	$(3)=[22.050\times(\frac{1}{2}+\frac{7}{8}+\frac{15}{16})]-[37,790\times(\frac{7}{8}+\frac{15}{16})]+[15,740\times\frac{15}{16}]$	$=2,800$
Resultant moment at	$(3)=\sqrt{50,300^2+2,800^2}$	$=50,400$

On referring to Table VII it will be found that a $2\frac{7}{8}$-inch pin, which has a bending value of 56,000 in.-lbs., is the size required.

Pin *C*. In Fig. 81 is shown one-half of pin *C* and the members which it connects. Since the vertical component of the stress in the tie bar is greater than the horizontal component, this member is placed as close to the vertical post as practicable, making allowance

for rivet heads flattened to $\frac{1}{4}$ in. high. The space between the tie bar and the top chord is filled by a cast-iron washer, or spool.

This pin is subject to its maximum moment when the bridge is loaded from d to h, in which case the stress in member $Cc = 32{,}650$ lbs., and that in $Cd = 40{,}810$ lbs., as shown in Fig. 77. The vertical and horizontal components of the stress in Cd are equal respectively to



Pin and Connected Members at C.

FIG. 81.

32,650 lbs. and 24,480 lbs. Thus the upward force at either bearing of post Cc, and the downward force at either tie bar are each equal to $32{,}650 \times \frac{1}{2} = 16{,}325$ lbs. Since the top chord is continuous at the pin, it is only the difference between the stresses in members BC and CD which need be considered; and this is evidently equal to the horizontal component of the stress in member Cd. Therefore, at either tie bar there is a force acting towards the right equal to $24{,}480 \times \frac{1}{2} = 12{,}240$ lbs., and at either bearing of the top chord, a force of equal magnitude acting towards the left.

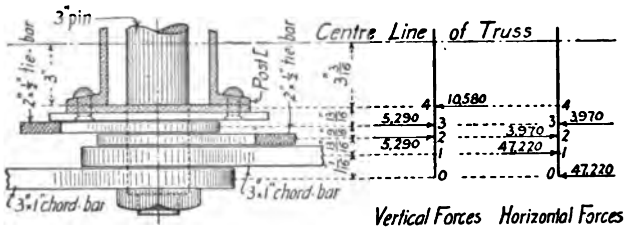
The vertical and horizontal forces acting on pin C are shown in Fig. 81, also the distances between them. Then

Vertical moment at	(1)	$= 0$
" "	$(2) = 16{,}325 \times \frac{3}{4}$	$= 12{,}240$
Horizontal moment at	$(1) = 12{,}240 \times 1\frac{1}{4}$	$= 15{,}300$
" "	$(2) = 12{,}240 \times (1\frac{1}{4} + \frac{3}{4} - \frac{3}{4})$	$= 15{,}300$
Result. moment at	$(2) = \sqrt{12{,}240^2 + 15{,}300^2}$	$= 19{,}600$

By Table VII a $2\frac{1}{2}$-inch pin, which has a bending value of 22,500 in.-lbs., is the size required.

Pin *e*. In Fig. 82 is shown one-half of pin *e* and the members connected thereby. The tie bars, as well as the chord bars, are outside of the post. In determining the distances c. to c. of these bars, $\frac{1}{16}$ in. is allowed for each bar to provide for the building up due to irregularities in the heads.

This pin is subject to its maximum moment when the bridge is fully loaded, in which case the stress in each chord bar is equal to $94{,}445 \times \frac{1}{2} = 47{,}220$ lbs. The vertical load on the pin-plates of the post is assumed to be equal to the total stress in the hip vertical, as shown in Fig. 77; thus the downward force at either bearing $= 21{,}150 \times \frac{1}{2} = 10{,}580$ lbs. The vertical component of the stress in the two tie bars shown is equal to the load on one pin-plate of the post; consequently,



Pin and Connected Members at e

FIG. 82.

the upward force at either tie bar $= 10{,}580 \times \frac{1}{2} = 5{,}290$ lbs. The horizontal component of the stress in each tie bar is equal to the vertical component, multiplied by the panel length and divided by the depth of the truss, $= 5{,}290 \times \frac{15}{20} = 3{,}970$ lbs. One of these forces acts towards the right, whereas the other acts towards the left.

The vertical and horizontal moments on this pin may now be computed, as follows:

Vertical moment at	(1) =		0
" "	(2) =		0
" "	(3) =	$5{,}290 \times \frac{9}{16}$	$= 3{,}000$
" "	(4) =	$5{,}290 \times (\frac{9}{16} + \frac{13}{16} + \frac{13}{16})$	$= 11{,}600$
Horizontal moment at	(1) =	$47{,}220 \times 1\frac{1}{16}$	$= 50{,}200$
" "	(2) =	$47{,}220 \times (1\frac{1}{16} + \frac{13}{16} - \frac{13}{16})$	$= 50{,}200$
" "	(3) =	$47{,}220 \times (1\frac{1}{16} + \frac{13}{16} + \frac{9}{16} - \frac{13}{16} - \frac{9}{16})] - [3{,}970 \times \frac{9}{16}]$	$= 48{,}000$
" "	(4) =		48,000
Resultant moment at	(4) =	$\sqrt{48{,}000^2 + 11{,}600^2}$	$= 49{,}400$

The maximum moment, which is at (1) and (2), $=50,200$ in.-lbs.; and, by Table VII, a $2\frac{5}{8}$-inch pin, having a bending value of 56,000 in.-lbs., is the size required.

Throughout the bottom chord and at the hips, 3-inch pins are used, while $2\frac{1}{2}$-inch are employed at the intermediate panel-points of the top chord. The pins are turned down at the ends (or shouldered), and provided with chambered, or recessed, nuts in order to insure full bearings for the outer members.*

Pin-Plates. The thickness of all bearings must be such that the pressure on the pins will not exceed 24,000 lbs. per sq.in.

In the end posts the total stress $=73,450$ lbs.; then $73,450 \div 24,000 = 3.06$ sq.ins. bearing area required on the pins; and, since the diameter of the pins $=3$ ins., the thickness of the bearings must be equal to at least $3.06 \div 3 = 1.02$ ins. Now the webs of the 9-inch channels at 20 lbs. are each $\frac{7}{16}$ in. thick, which are reinforced at *a* and *B* by two $\frac{3}{8}$-inch pin-plates, making the total thickness of bearing at these points $(\frac{7}{16}+\frac{3}{8}) \times 2 = 1\frac{5}{8}$ ins. The load on the pin-plates will be equal to the total stress in the post, multiplied by the thickness of these pin-plates and divided by the total thickness of the bearing, $=73,450 \times \frac{0.75}{1.625} = 34,000$ lbs.; and (since the single shearing value of one $\frac{3}{4}$-inch rivet $=5,300$ lbs.), $34,000 \div 5,300 = 6.4$ rivets required in the two plates at either joint. The detail drawing, Fig. 78, shows six rivets in each plate, being nearly twice as many as required to meet the stress.

In the top chord section *BC*, the total stress $=75,580$ lbs.; then $75,580 \div 24,000 = 3.15$ sq.ins. bearing area required on the pin at *B*; and $3.15 \div 3 = 1.05$ ins., which is the thickness of bearing required. The webs of the 9-inch channels at 15 lbs. are each $\frac{5}{16}$ in. thick, and there are two $\frac{3}{8}$-inch pin-plates, making the total thickness of bearing $(\frac{5}{16}+\frac{3}{8}) \times 2 = 1\frac{3}{8}$ ins. The load on the pin-plates $=75,580 \times \frac{0.75}{1.375} = 41,300$ lbs.; and $41,300 \div 5,300 = 7.8$ rivets required. The drawing shows six rivets in each pin-plate, as before.

When the pin-plates extend beyond the member reinforced thereby, as at *a* and *B*, they are usually termed *jaw plates*. At panel-point *B*, these plates are on the outside of the end post, but on the inside of the top chord.

At panel-point *C*, the top chord is continuous; and it is only necessary to provide sufficient bearing for the horizontal component

of the stress in the tie bar Cd, which is equal to $40,810\times\frac{15}{25}=24,500$ lbs.; then $24,500\div 24,000=1.02$ sq.ins. bearing area required on the pin; and $1.02\div 2.5=0.41$ in., the thickness of bearing required. The webs of the two 9-inch channels at 20 lbs. are together equal to $\frac{7}{8}$ in., thus no pin-plates are required.

For the hip vertical tension member Bb, the pin-plates at B should be proportioned so that the net area between the upper edge of the pin hole and the top of the pin-plates will be at least equal to that required in the member itself; and that the net area through the pin hole will be 50 per cent greater than required in the member. Now the net sectional area required in the body of this member, as given in Fig. 77, $=1.32$ sq.ins.; and, since the pin-plates are $\frac{3}{8}$ in. thick, and the distance from the centre of the 3-inch pin to the top of the plates is 4 ins., the net area back of the pin hole (or above it) $=(4-1\frac{1}{2})\times\frac{3}{4}=1.875$ sq.ins. The net sectional area required through the pin hole $=1.32\times 1\frac{1}{2}=1.98$ sq.ins.; and, since the pin-plates are 9 ins. wide, the net area through the pin hole $=(9-3)\times\frac{3}{4}=4.5$ sq.ins. The rivets used in this member are $\frac{5}{8}$ in. diameter, the single shearing value, by Table V, being 3,680 lbs.; and, since the total stress $=21,150$ lbs. the number of rivets required in the pin-plates $=21,150\div 3,680=6$; whereas the drawing shows six rivets in each plate—or twelve rivets in all.

In post Cc, the total stress $=32,650$ lbs.; then, for the pin C, the bearing area required $=32,650\div 24,000=1.36$ sq.ins.; and the thickness of bearing required $=1.36\div 2.5=0.54$ in. There are two $\frac{3}{8}$-inch plates provided, making a total thickness of $\frac{3}{4}$ in. The number of $\frac{5}{8}$-inch rivets required in these plates $=32,650\div 3,680=9$, while twelve rivets are provided, as shown. At pin c, the bearing is greater than the stress in the post on account of the additional load from the floor beam at this point; thus the maximum bearing on the pin is equal to the vertical component of the stress in tie bar $Bc=55,400\times\frac{20}{25}=44,300$ lbs., which, at 24,000, $=1.85$ sq.ins. required; and the required thickness of bearing $=1.85\div 3=0.62$ in.; whereas the total thickness of the two $\frac{3}{8}$-inch plates $=\frac{3}{4}$ in.

The 6-inch channels of all the vertical members, as well as the inside pin-plates, are extended below the bottom chord for the connection of the floorbeams; whereas the outside pin-plates are only 9 ins. long, as shown.

Latticing. The latticing of the chords and posts is proportioned in accordance with rules given in Art. 16, Chap. IV, viz.: Single

latticing should make an angle of 60° with the longitudinal axis of the column; the width of the lattice bars should not be less than two and one-half times the diameter of the rivets used therein; and their thickness should not be less than one-fortieth of their length c. to c. of rivets.

In the end posts and top chords, the rivets are $\frac{3}{4}$ in. diameter; thus the minimum width of the lattice bars may be $2\frac{1}{2}\times\frac{3}{4}=1\frac{7}{8}$ ins., whereas the bars used are $2\frac{1}{4}\times\frac{3}{8}$ ins. The length of the bars c. to c. of rivets is about $14\frac{3}{4}$ ins., so their minimum thickness may be $14.75\div 40=0.37$ in. Probably the lattice bars subject to the greatest stress are those in the upper part of the end posts, where the shear due to the wind-load is equal to the induced force at O or M (Fig. 77) $=4,266$ lbs.; and, since the post is latticed on two sides, the bars and connecting rivets are evidently amply strong for this or any other duty they may be called upon to perform. As an exercise, the student may also investigate the efficiency of the lattice bars in accordance with the theory set forth in Art. 16.

In the vertical posts, the rivets are $\frac{5}{8}$ in. diameter; so the minimum width of the lattice bars may be $2\frac{1}{2}\times\frac{5}{8}=1\frac{9}{16}$ ins., whereas the bars used are $2\times\frac{5}{16}$ ins. The length of the bars c. to c. of rivets is about $8\frac{1}{2}$ ins.; thus the minimum thickness may be $8.5\div 40=0.21$ in.

Top Chord Splices. The top chord is spliced at points 9 ins. from pins C, D, and F. Since these are faced joints, the stresses are transmitted directly by the abutting surfaces, and the splice-plates are required only to hold the members in line. At panel-points B and H, the stresses in the plane of the truss are assumed to be transmitted through the pins, while the bent plates on the upper and lower flanges of the channels are only counted on to take care of the lateral stresses.

Pier Members. The pier members consist of the shoes, rollers, bed-plates, and anchor bolts.

The total load on the shoe is equal to the vertical component of the stress in the end post, $=73,450\times\frac{20}{25}=58,700$ lbs. Thus the bearing area required for the pin on the shoe standards$=58,700\div 24,000$ $=2.44$ sq.ins.; and the required thickness of bearing$=2.44\div 3=0.81$ in. Two $7\times3\frac{1}{2}\times\frac{5}{8}$ in. angles are used, their combined thickness being $1\frac{1}{4}$ ins. The object in using heavier angles in this place than required for pin bearing alone is to secure additional stiffness transversely.

Assuming the rollers to be 3 ins. in diameter, the permissible load per lineal inch$=1,200\sqrt{3}=2,080$ lbs.; and $58,700\div 2,080=28$ lin.ins.

required. There are four rollers, the bearing length of each being $(15\frac{1}{2}-2)=13\frac{1}{2}$ ins., thus the total length of bearing provided $=13\frac{1}{2}\times 4 = 54$ ins. The rollers are turned down at the centre for a length of 2 ins. in order to engage in the guide strips on the shoe- and bed-plates; and the ends are turned down to enter holes in the $2\frac{1}{2}\times\frac{3}{8}$ in. spacing bars, as shown. The ends of the outer rollers are made long enough to insert cotter-pins, whereas the ends of the intermediate rollers are only long enough to pass through the spacing bars.

At the expansion end, the shoe- and bed-plates are each $\frac{7}{8}$ in. thick; the former planed on the bottom and the latter on the top, to $\frac{3}{4}$ in. thick, leaving strips $1\frac{7}{8}$ ins. wide on the longitudinal centre line to serve as roller guides. The required area of the bed-plates $=58,700 \div 400 = 147$ sq.ins.; but the shoe- and bed-plates must also be large enough to accommodate the rollers and anchor bolts. The area of the bed-plates used $=15\times 22=330$ sq.ins. The shoe-plates are extended, as shown, to provide connections for the bottom laterals. Slotted holes $1\frac{3}{8}\times 3$ ins. are provided in the roller shoe-plates and round holes $1\frac{3}{8}$ ins. diameter in the bed-plates for the $1\frac{1}{4}$ in. anchor bolts.

At the fixed end, the bed-plates are made of cast iron, and their height is equal to the diameter of the rollers, plus the thickness of the expansion end bed-plate $=3\frac{3}{4}$ ins. With this arrangement, both bridge seats may be built at the same elevation.

Eye-Bars and Rods. The heads of the steel eye-bars are made by upsetting the ends of the bars, and forging in dies of the required size. These heads are circular in form, and made of such diameter that the sectional area through the pin hole is 50 per cent greater than that of the bar. Thus, for a bar 3 ins. wide with 3 ins. diameter pin holes, the diameter of the heads $=(3\times 1\frac{1}{2})+3=7\frac{1}{2}$ ins.

The loop ends of the square iron bars, used for counter ties, are made by bending the ends of a bar around a pin of the required size, and welding these ends to the body of the bar. The distance from the centre of the pin hole to the crotch should not be less than two and one-half times the diameter of the pin. These members are made in two parts and provided with turnbuckles for adjustment, as shown.

Estimated Weight. An estimate, made from the detail drawing, Fig. 78, shows that the metal in the bridge weighs about 54,000 lbs. Then $54,000 \div 120 = 450$ lbs. per lin.ft.; whereas the weight of steel assumed for calculating the dead-load stresses was only 375 lbs. per

lin.ft. The extra weight is principally due to the lateral and sway bracing which are exceptionally heavy for a bridge of this class. It would be a profitable exercise for the student to recalculate the trusses in accordance with the corrected dead-load; but it will be found to make little difference in the total stresses, and probably none in the resulting sections.

CHAPTER XII

Coefficients for Stresses in Various Types of Trusses

A large percentage of the trusses used for highway bridges are constructed with parallel chords and designed for uniform loads. Now the labor of computing the stresses in these trusses may be greatly reduced by employing the tables of coefficients given herewith. In each of the tables, the dead- and live-loads are assumed to be uniform, and concentrated at the panel-points of the loaded chord.

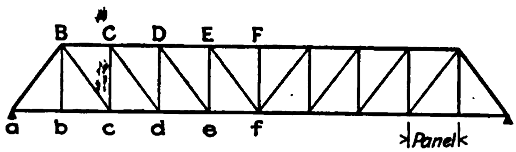
In connection with the Pratt trusses (Tables VIII and IX), it will be noted that the live-load tends to induce in most of the web members compression as well as tension. Thus, when the compression due to the live-load exceeds the tension due to the dead-load, the member should be designed as a column, otherwise a counter tie must be provided, and the tension therein will be equal to the difference between the dead-load tension in the main tie and the live-load compression when no counter ties are used.

In the Warren girders (Tables X and XI), as well as in the subdivided lattice girders (Table XII), no counter ties are employed; thus the members subject to compression from the live-load exceeding the tension due to the dead-load must be proportioned as columns.

In all cases, the sign + indicates compression, and the sign –, tension. *DL* represents dead-load and *LL* live-load.

Table VIII

COEFFICIENTS FOR MAXIMUM STRESSES IN THROUGH PRATT TRUSSES



Members.	10 Panels.		9 Panels.		8 Panels.		7 Panels.		6 Panels.	
	DL	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*
aB	$+\frac{45}{10}$	$+\frac{45}{10}$	$+\frac{36}{9}$	$+\frac{36}{9}$	$+\frac{28}{8}$	$+\frac{28}{8}$	$+\frac{21}{7}$	$+\frac{21}{7}$	$+\frac{15}{6}$	$+\frac{15}{6}$
Bb	$-\frac{10}{10}$	$-\frac{10}{10}$	$-\frac{9}{9}$	$-\frac{9}{9}$	$-\frac{8}{8}$	$-\frac{8}{8}$	$-\frac{7}{7}$	$-\frac{7}{7}$	$-\frac{6}{6}$	$-\frac{6}{6}$
Bc	$-\frac{35}{10}$	$-\frac{36}{10}$ $+\frac{1}{10}$	$-\frac{27}{9}$	$-\frac{28}{9}$ $+\frac{1}{9}$	$-\frac{20}{8}$	$-\frac{21}{8}$ $+\frac{1}{8}$	$-\frac{14}{7}$	$-\frac{15}{7}$ $+\frac{1}{7}$	$-\frac{9}{6}$	$-\frac{10}{6}$ $+\frac{1}{6}$
Cc	$+\frac{25}{10}$	$+\frac{28}{10}$ $-\frac{3}{10}$	$+\frac{18}{9}$	$+\frac{21}{9}$ $-\frac{3}{9}$	$+\frac{12}{8}$	$+\frac{15}{8}$ $-\frac{3}{8}$	$+\frac{7}{7}$	$+\frac{10}{7}$ $-\frac{3}{7}$	$+\frac{8}{6}$	$+\frac{6}{6}$ $-\frac{3}{6}$
Cd	$-\frac{25}{10}$	$-\frac{28}{10}$ $+\frac{3}{10}$	$-\frac{18}{9}$	$-\frac{21}{9}$ $+\frac{3}{9}$	$-\frac{12}{8}$	$-\frac{15}{8}$ $+\frac{3}{8}$	$-\frac{7}{7}$	$-\frac{10}{7}$ $+\frac{3}{7}$	$-\frac{3}{6}$	$-\frac{6}{6}$ $+\frac{3}{6}$
Dd	$+\frac{15}{10}$	$+\frac{21}{10}$ $-\frac{6}{10}$	$+\frac{9}{9}$	$+\frac{15}{9}$ $-\frac{6}{9}$	$+\frac{4}{8}$	$+\frac{10}{8}$ $-\frac{6}{8}$	0	$+\frac{6}{7}$ $-\frac{6}{7}$	0	0
De	$-\frac{15}{10}$	$-\frac{21}{10}$ $+\frac{6}{10}$	$-\frac{9}{9}$	$-\frac{15}{9}$ $+\frac{6}{9}$	$-\frac{4}{8}$	$-\frac{10}{8}$ $+\frac{6}{8}$	0	$-\frac{6}{7}$ $+\frac{6}{7}$		
Ee	$+\frac{5}{10}$	$+\frac{15}{10}$ $-\frac{10}{10}$	0	$+\frac{10}{9}$ $-\frac{10}{9}$	0	0				
Ef	$-\frac{5}{10}$	$-\frac{15}{10}$ $+\frac{10}{10}$	0	$-\frac{10}{10}$ $+\frac{10}{9}$						
Ff	0	0								

TABLE VIII—*Continued*

Members.	10 Panels.		9 Panels.		8 Panels.		7 Panels.		6 Panels.	
	DL	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*
abc	$-\frac{45}{10}$	$-\frac{45}{10}$	$-\frac{36}{9}$	$-\frac{36}{9}$	$-\frac{28}{8}$	$-\frac{28}{8}$	$-\frac{21}{7}$	$-\frac{21}{7}$	$-\frac{15}{6}$	$-\frac{15}{6}$
cd	$-\frac{80}{10}$	$-\frac{80}{10}$	$-\frac{63}{9}$	$-\frac{63}{9}$	$-\frac{48}{8}$	$-\frac{48}{8}$	$-\frac{35}{7}$	$-\frac{35}{7}$	$-\frac{24}{6}$	$-\frac{24}{6}$
de	$-\frac{105}{10}$	$-\frac{105}{10}$	$-\frac{81}{9}$	$-\frac{81}{9}$	$-\frac{60}{8}$	$-\frac{60}{7}$	$-\frac{42}{7}$	$-\frac{42}{7}$		
ef	$-\frac{120}{10}$	$-\frac{120}{10}$	$-\frac{90}{9}$	$-\frac{90}{9}$						
BC	$+\frac{80}{10}$	$+\frac{80}{10}$	$+\frac{63}{9}$	$+\frac{63}{9}$	$+\frac{48}{8}$	$+\frac{48}{8}$	$+\frac{35}{7}$	$+\frac{35}{7}$	$+\frac{21}{6}$	$+\frac{24}{6}$
CD	$+\frac{105}{10}$	$+\frac{105}{10}$	$+\frac{81}{9}$	$+\frac{81}{9}$	$+\frac{60}{8}$	$+\frac{60}{8}$	$+\frac{42}{7}$	$+\frac{42}{7}$	$+\frac{27}{6}$	$+\frac{27}{6}$
DE	$+\frac{120}{10}$	$+\frac{120}{10}$	$+\frac{90}{9}$	$+\frac{90}{9}$	$+\frac{64}{8}$	$+\frac{64}{8}$	$+\frac{42}{7}$	$+\frac{42}{7}$		
EF	$+\frac{125}{10}$	$+\frac{125}{10}$	$+\frac{90}{9}$	$+\frac{90}{9}$						

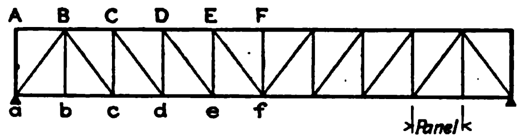
Stresses in diagonal members = panel load $\times$ coefficient $\times \dfrac{\text{length of member}}{\text{depth of truss}}$.

Stresses in vertical members = panel load $\times$ coefficient $\times$ 1.

Stresses in chord members = panel load $\times$ coefficient $\times \dfrac{\text{length of panel}}{\text{depth of truss}}$.

Table IX

COEFFICIENTS FOR MAXIMUM STRESSES IN DECK PRATT TRUSSES



Members.	10 Panels.		9 Panels.		8 Panels.		7 Panels.		6 Panels.	
	DL	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*
Aa	$+\frac{5}{10}$	$+\frac{5}{10}$	$+\frac{4.5}{9}$	$+\frac{4.5}{9}$	$+\frac{4}{8}$	$+\frac{4}{8}$	$+\frac{3.5}{7}$	$+\frac{3.5}{7}$	$+\frac{3}{6}$	$+\frac{3}{6}$
aB	$+\frac{45}{10}$	$+\frac{45}{10}$	$+\frac{36}{9}$	$+\frac{36}{9}$	$+\frac{28}{8}$	$+\frac{28}{8}$	$+\frac{21}{7}$	$+\frac{21}{7}$	$+\frac{15}{6}$	$+\frac{15}{6}$
Bb	0	0	0	0	0	0	0	0	0	0
Bc	$-\frac{35}{10}$	$-\frac{36}{10}$ $+\frac{1}{10}$	$-\frac{27}{9}$	$-\frac{28}{9}$ $+\frac{1}{9}$	$-\frac{20}{8}$	$-\frac{21}{8}$ $+\frac{1}{8}$	$-\frac{14}{7}$	$-\frac{15}{7}$ $+\frac{1}{7}$	$-\frac{9}{6}$	$-\frac{10}{6}$ $+\frac{1}{6}$
Cc	$+\frac{35}{10}$	$+\frac{36}{10}$ $-\frac{1}{10}$	$+\frac{27}{9}$	$+\frac{28}{9}$ $-\frac{1}{9}$	$+\frac{20}{8}$	$+\frac{21}{8}$ $-\frac{1}{8}$	$+\frac{14}{7}$	$+\frac{15}{7}$ $-\frac{1}{7}$	$+\frac{9}{6}$	$+\frac{10}{6}$ $-\frac{1}{6}$
Cd	$-\frac{25}{10}$	$-\frac{28}{10}$ $+\frac{3}{10}$	$-\frac{18}{9}$	$-\frac{21}{9}$ $+\frac{3}{9}$	$-\frac{12}{8}$	$-\frac{15}{8}$ $+\frac{3}{8}$	$-\frac{7}{7}$	$-\frac{10}{7}$ $+\frac{3}{7}$	$-\frac{3}{6}$	$-\frac{6}{6}$ $+\frac{3}{6}$
Dd	$+\frac{25}{10}$	$+\frac{28}{10}$ $-\frac{3}{10}$	$+\frac{18}{9}$	$+\frac{21}{9}$ $-\frac{3}{9}$	$+\frac{12}{8}$	$+\frac{15}{8}$ $-\frac{3}{8}$	$+\frac{7}{7}$	$+\frac{10}{7}$ $-\frac{3}{7}$	$+\frac{6}{6}$	$+\frac{6}{6}$
De	$-\frac{15}{10}$	$-\frac{21}{10}$ $+\frac{6}{10}$	$-\frac{9}{9}$	$-\frac{15}{9}$ $+\frac{6}{9}$	$-\frac{4}{8}$	$-\frac{10}{8}$ $+\frac{6}{8}$	0	$-\frac{6}{7}$ $+\frac{6}{7}$		
Ee	$+\frac{15}{10}$	$+\frac{21}{10}$ $-\frac{6}{10}$	$+\frac{9}{9}$	$+\frac{15}{9}$ $-\frac{6}{9}$	$+\frac{8}{8}$	$+\frac{8}{8}$				
Ef	$-\frac{5}{10}$	$-\frac{15}{10}$ $+\frac{10}{10}$	0	$-\frac{10}{9}$ $+\frac{10}{9}$						
Ff	$+\frac{10}{10}$	$+\frac{10}{10}$								

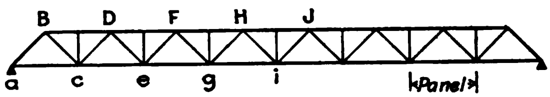
TABLE IX—*Continued*

Members.	10 Panels.		9 Panels.		8 Panels.		7 Panels.		6 Panels.	
	DL	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*
abc	$-\frac{45}{10}$	$-\frac{45}{10}$	$-\frac{36}{9}$	$-\frac{36}{9}$	$-\frac{28}{8}$	$-\frac{28}{8}$	$-\frac{21}{7}$	$-\frac{21}{7}$	$-\frac{15}{6}$	$-\frac{15}{6}$
cd	$-\frac{80}{10}$	$-\frac{80}{10}$	$-\frac{63}{9}$	$-\frac{63}{9}$	$-\frac{48}{8}$	$-\frac{48}{8}$	$-\frac{35}{7}$	$-\frac{35}{7}$	$-\frac{24}{6}$	$-\frac{24}{6}$
de	$-\frac{105}{10}$	$-\frac{105}{10}$	$-\frac{81}{9}$	$-\frac{81}{9}$	$-\frac{60}{8}$	$-\frac{60}{8}$	$-\frac{42}{7}$	$-\frac{42}{7}$		
ef	$-\frac{120}{10}$	$-\frac{120}{10}$	$-\frac{90}{9}$	$-\frac{90}{9}$						
AB	0	0	0	0	0	0	0	0	0	0
BC	$+\frac{80}{10}$	$+\frac{80}{10}$	$+\frac{63}{9}$	$+\frac{63}{9}$	$+\frac{48}{8}$	$+\frac{48}{8}$	$+\frac{35}{7}$	$+\frac{35}{7}$	$+\frac{24}{6}$	$+\frac{24}{6}$
CD	$+\frac{105}{10}$	$+\frac{105}{10}$	$+\frac{81}{9}$	$+\frac{81}{9}$	$+\frac{60}{8}$	$+\frac{60}{8}$	$+\frac{42}{7}$	$+\frac{42}{7}$	$+\frac{27}{6}$	$+\frac{27}{6}$
DE	$+\frac{120}{10}$	$+\frac{120}{10}$	$+\frac{90}{9}$	$+\frac{90}{9}$	$+\frac{64}{8}$	$+\frac{64}{8}$	$+\frac{42}{7}$	$+\frac{42}{7}$		
EF	$+\frac{125}{10}$	$+\frac{125}{10}$	$+\frac{90}{9}$	$+\frac{90}{9}$						

Stresses in diagonal members = panel load $\times$ coefficient $\times \frac{\text{length of member}}{\text{depth of truss}}$.
Stresses in vertical members = panel load $\times$ coefficient $\times$ 1.
Stresses in chord members = panel load $\times$ coefficient $\times \frac{\text{length of panel}}{\text{depth of truss}}$.

Table X

COEFFICIENTS FOR MAXIMUM STRESSES IN THROUGH WARREN GIRDERS



Members.	8 Panels.		7 Panels.		6 Panels.		5 Panels.		4 Panels.	
	DL	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*
aB	$+\frac{28}{8}$	$+\frac{28}{8}$	$+\frac{21}{7}$	$+\frac{21}{7}$	$+\frac{15}{6}$	$+\frac{15}{6}$	$+\frac{10}{5}$	$+\frac{10}{5}$	$+\frac{6}{4}$	$+\frac{6}{4}$
Bc	$-\frac{28}{8}$	$-\frac{28}{8}$	$-\frac{21}{7}$	$-\frac{21}{7}$	$-\frac{15}{6}$	$-\frac{15}{6}$	$-\frac{10}{5}$	$-\frac{10}{5}$	$-\frac{6}{4}$	$-\frac{6}{4}$
cD	$+\frac{20}{8}$	$+\frac{21}{8}$ $-\frac{1}{8}$	$+\frac{14}{7}$	$+\frac{15}{7}$ $-\frac{1}{7}$	$+\frac{9}{6}$	$+\frac{10}{6}$ $-\frac{1}{6}$	$+\frac{5}{5}$	$+\frac{6}{5}$ $-\frac{1}{5}$	$+\frac{2}{4}$	$+\frac{3}{4}$ $-\frac{1}{4}$
De	$-\frac{20}{8}$	$-\frac{21}{8}$ $+\frac{1}{8}$	$-\frac{14}{7}$	$-\frac{15}{7}$ $+\frac{1}{7}$	$-\frac{9}{6}$	$-\frac{10}{6}$ $+\frac{1}{6}$	$-\frac{5}{5}$	$-\frac{6}{5}$ $+\frac{1}{5}$	$-\frac{2}{4}$	$-\frac{3}{4}$ $+\frac{1}{4}$
eF	$+\frac{12}{8}$	$+\frac{15}{8}$ $-\frac{3}{8}$	$+\frac{7}{7}$	$+\frac{10}{7}$ $-\frac{3}{7}$	$+\frac{3}{6}$	$+\frac{6}{6}$ $-\frac{3}{6}$	0	$+\frac{3}{5}$ $-\frac{3}{5}$		
Fg	$-\frac{12}{8}$	$-\frac{15}{8}$ $+\frac{3}{8}$	$-\frac{7}{7}$	$-\frac{10}{7}$ $+\frac{3}{7}$	$-\frac{3}{6}$	$-\frac{6}{6}$ $+\frac{3}{3}$				
gH	$+\frac{4}{8}$	$+\frac{10}{8}$ $-\frac{6}{8}$	0	$+\frac{6}{7}$ $-\frac{6}{7}$						
Hi	$-\frac{4}{8}$	$-\frac{10}{8}$ $+\frac{6}{8}$								
ac	$-\frac{14}{8}$	$-\frac{14}{8}$	$-\frac{10.5}{7}$	$-\frac{10.5}{7}$	$-\frac{7.5}{6}$	$-\frac{7.5}{6}$	$-\frac{5}{5}$	$-\frac{5}{5}$	$-\frac{3}{4}$	$-\frac{3}{4}$
ce	$-\frac{38}{8}$	$-\frac{38}{8}$	$-\frac{28}{7}$	$-\frac{28}{7}$	$-\frac{19.5}{6}$	$-\frac{19.5}{6}$	$-\frac{12.5}{5}$	$-\frac{12.5}{5}$	$-\frac{7}{4}$	$-\frac{7}{4}$
eg	$-\frac{54}{8}$	$-\frac{54}{8}$	$-\frac{38.5}{7}$	$-\frac{38.5}{7}$	$-\frac{25.5}{6}$	$-\frac{25.5}{6}$	$-\frac{15}{5}$	$-\frac{15}{5}$		
gi	$-\frac{62}{8}$	$-\frac{62}{8}$	$-\frac{42}{7}$	$-\frac{42}{7}$						

TABLE X—*Continued*

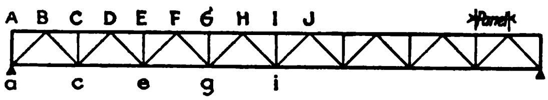
Members.	8 Panels.		7 Panels.		6 Panels.		5 Panels.		4 Panels.	
	DL	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*
BD	$+\frac{28}{8}$	$+\frac{28}{8}$	$+\frac{21}{7}$	$+\frac{21}{6}$	$+\frac{15}{6}$	$+\frac{15}{6}$	$+\frac{10}{5}$	$+\frac{10}{5}$	$+\frac{6}{4}$	$+\frac{6}{4}$
DF	$+\frac{48}{8}$	$+\frac{48}{8}$	$+\frac{35}{7}$	$+\frac{35}{7}$	$+\frac{24}{6}$	$+\frac{24}{6}$	$+\frac{15}{5}$	$+\frac{15}{5}$	$+\frac{8}{4}$	$+\frac{8}{4}$
FH	$+\frac{60}{8}$	$+\frac{60}{}$	$+\frac{42}{7}$	$+\frac{42}{7}$	$+\frac{27}{6}$	$+\frac{27}{6}$				
HJ	$+\frac{64}{8}$	$+\frac{64}{8}$								

Stresses in diagonal members = panel load × coefficient × $\frac{\text{length of member}}{\text{depth of truss}}$.

Stresses in chord members = panel load × coefficient × $\frac{\text{length of panel}}{\text{depth of truss}}$.

Table XI

COEFFICIENTS FOR MAXIMUM STRESSES IN DECK WARREN GIRDERS



Members.	16 Panels.		14 Panels.		12 Panels.		10 Panels.		8 Panels.	
	DL	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*
Aa	$+\frac{8}{16}$	$+\frac{8}{16}$	$+\frac{7}{14}$	$+\frac{7}{14}$	$+\frac{6}{12}$	$+\frac{6}{12}$	$+\frac{5}{10}$	$+\frac{5}{10}$	$+\frac{4}{8}$	$+\frac{4}{8}$
Cc	$+\frac{16}{16}$	$+\frac{16}{16}$	$+\frac{14}{14}$	$+\frac{14}{14}$	$+\frac{12}{12}$	$+\frac{12}{12}$	$+\frac{10}{10}$	$+\frac{10}{10}$	$+\frac{8}{8}$	$+\frac{8}{8}$
aB	$+\frac{120}{16}$	$+\frac{120}{16}$	$+\frac{91}{14}$	$+\frac{91}{14}$	$+\frac{66}{12}$	$+\frac{66}{12}$	$+\frac{45}{10}$	$+\frac{45}{10}$	$+\frac{28}{8}$	$+\frac{28}{8}$
Bc	$-\frac{104}{16}$	$-\frac{105}{16}$ $+\frac{1}{16}$	$-\frac{77}{14}$	$-\frac{78}{14}$ $+\frac{1}{14}$	$-\frac{54}{12}$	$-\frac{55}{12}$ $+\frac{1}{12}$	$-\frac{35}{10}$	$-\frac{36}{10}$ $+\frac{1}{10}$	$-\frac{20}{8}$	$-\frac{21}{8}$ $+\frac{1}{8}$
cD	$+\frac{88}{16}$	$+\frac{91}{16}$ $-\frac{3}{16}$	$+\frac{63}{14}$	$+\frac{66}{14}$ $-\frac{3}{14}$	$+\frac{42}{12}$	$+\frac{45}{12}$ $-\frac{3}{12}$	$+\frac{25}{10}$	$+\frac{28}{10}$ $-\frac{3}{10}$	$+\frac{12}{8}$	$+\frac{15}{8}$ $-\frac{3}{8}$
De	$-\frac{72}{16}$	$-\frac{78}{16}$ $+\frac{6}{16}$	$-\frac{49}{14}$	$-\frac{55}{14}$ $+\frac{6}{14}$	$-\frac{30}{12}$	$-\frac{36}{12}$ $+\frac{6}{12}$	$-\frac{15}{10}$	$-\frac{21}{10}$ $+\frac{6}{10}$	$-\frac{4}{8}$	$-\frac{10}{8}$ $+\frac{6}{8}$
eF	$+\frac{56}{16}$	$+\frac{66}{16}$ $-\frac{10}{16}$	$+\frac{35}{14}$	$+\frac{45}{14}$ $-\frac{10}{14}$	$+\frac{18}{12}$	$+\frac{28}{12}$ $-\frac{10}{12}$	$+\frac{5}{10}$	$+\frac{15}{10}$ $-\frac{10}{10}$		
Fg	$-\frac{40}{16}$	$-\frac{55}{16}$ $+\frac{15}{16}$	$-\frac{21}{14}$	$-\frac{36}{14}$ $+\frac{15}{14}$	$-\frac{6}{12}$	$-\frac{21}{12}$ $+\frac{15}{12}$				
gH	$+\frac{24}{16}$	$+\frac{45}{16}$ $-\frac{21}{16}$	$+\frac{7}{14}$	$+\frac{28}{14}$ $-\frac{21}{14}$						
Hi	$-\frac{8}{16}$	$-\frac{36}{16}$ $+\frac{28}{16}$								
ac	$-\frac{120}{16}$	$-\frac{120}{16}$	$-\frac{91}{14}$	$-\frac{91}{14}$	$-\frac{66}{12}$	$-\frac{66}{12}$	$-\frac{45}{10}$	$-\frac{45}{10}$	$-\frac{28}{8}$	$-\frac{28}{8}$

TABLE XI—*Continued*

Members.	16 Panels.		14 Panels.		12 Panels.		10 Panels.		8 Panels.	
	DL	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*
ce	$-\frac{312}{16}$	$-\frac{312}{16}$	$-\frac{231}{14}$	$-\frac{231}{14}$	$-\frac{162}{12}$	$-\frac{162}{12}$	$-\frac{105}{10}$	$-\frac{105}{10}$	$-\frac{60}{8}$	$-\frac{60}{8}$
eg	$-\frac{440}{16}$	$-\frac{440}{16}$	$-\frac{315}{14}$	$-\frac{315}{14}$	$-\frac{210}{12}$	$-\frac{210}{12}$	$-\frac{125}{10}$	$-\frac{125}{10}$		
gi	$-\frac{504}{16}$	$-\frac{504}{16}$	$-\frac{343}{14}$	$-\frac{343}{14}$						
AB	0	0	0	0	0	0	0	0	0	0
BCD	$+\frac{224}{16}$	$+\frac{224}{16}$	$+\frac{168}{14}$	$+\frac{168}{14}$	$+\frac{120}{12}$	$+\frac{120}{12}$	$+\frac{80}{10}$	$+\frac{80}{10}$	$+\frac{48}{8}$	$+\frac{48}{8}$
DEF	$+\frac{384}{16}$	$+\frac{384}{16}$	$+\frac{280}{14}$	$+\frac{280}{14}$	$+\frac{192}{12}$	$+\frac{192}{12}$	$+\frac{120}{10}$	$+\frac{120}{10}$	$+\frac{64}{8}$	$+\frac{64}{8}$
FGH	$+\frac{480}{16}$	$+\frac{480}{16}$	$+\frac{336}{14}$	$+\frac{336}{14}$	$+\frac{216}{12}$	$+\frac{216}{12}$				
HIJ	$+\frac{512}{16}$	$+\frac{512}{16}$								

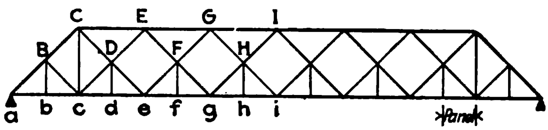
Stresses in diagonals member = panel load $\times$ coefficient $\times \frac{\text{length of member}}{\text{depth of truss}}$.

Stresses in vertical members = panel load $\times$ coefficient $\times$ 1.

Stresses in chord members = panel load $\times$ coefficient $\times \frac{\text{length of panel}}{\text{depth of truss}}$.

TABLE XII

COEFFICIENTS FOR MAXIMUM STRESSES IN THROUGH SUBDIVIDED LATTICE GIRDERS



Members	16 Panels.		14 Panels.		12 Panels.		10 Panels.		8 Panels.	
	DL	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*
aB	$+\frac{120}{16}$	$+\frac{120}{16}$	$+\frac{91}{14}$	$+\frac{91}{14}$	$+\frac{66}{12}$	$+\frac{66}{12}$	$+\frac{45}{10}$	$+\frac{45}{10}$	$+\frac{28}{8}$	$+\frac{28}{8}$
BC	$+\frac{112}{16}$	$+\frac{112}{16}$	$+\frac{84}{14}$	$+\frac{84}{14}$	$+\frac{60}{12}$	$+\frac{60}{12}$	$+\frac{40}{10}$	$+\frac{40}{10}$	$+\frac{24}{8}$	$+\frac{24}{8}$
CD	$-\frac{48}{16}$	$-\frac{49.5}{16}$ $+\frac{1.5}{16}$	$-\frac{33.5}{14}$	$-\frac{35}{14}$ $+\frac{1.5}{14}$	$-\frac{24}{12}$	$-\frac{25.5}{12}$ $+\frac{1.5}{12}$	$-\frac{13.5}{10}$	$-\frac{15}{10}$ $+\frac{1.5}{10}$	$-\frac{8}{8}$	$-\frac{9.5}{8}$ $+\frac{1.5}{8}$
De	$-\frac{40}{16}$	$-\frac{43}{16}$ $+\frac{3}{16}$	$-\frac{26.5}{14}$	$-\frac{29.5}{14}$ $+\frac{3}{14}$	$-\frac{18}{12}$	$-\frac{21}{12}$ $+\frac{3}{12}$	$-\frac{8.5}{10}$	$-\frac{11.5}{10}$ $+\frac{3}{10}$	$-\frac{4}{8}$	$-\frac{7}{8}$ $+\frac{3}{8}$
eF	$+\frac{24}{16}$	$+\frac{31}{16}$ $-\frac{7}{16}$	$+\frac{12.5}{14}$	$+\frac{19.5}{14}$ $-\frac{7}{14}$	$+\frac{6}{12}$	$+\frac{13}{12}$ $-\frac{7}{12}$	$-\frac{1.5}{10}$	$-\frac{7}{10}$ $+\frac{5.5}{10}$		
FG	$+\frac{16}{16}$	$+\frac{25.5}{16}$ $-\frac{9.5}{16}$	$+\frac{5.5}{14}$	$+\frac{15}{14}$ $-\frac{9.5}{14}$	0	$+\frac{9.5}{12}$ $-\frac{9.5}{12}$				
GH	$-\frac{16}{16}$	$-\frac{25.5}{16}$ $+\frac{9.5}{16}$	$-\frac{5.5}{14}$	$-\frac{15}{14}$ $+\frac{9.5}{14}$						
Hi	$-\frac{8}{16}$	$-\frac{21}{16}$ $+\frac{13}{16}$								
Bc	$+\frac{8}{16}$	$+\frac{8}{16}$	$+\frac{7}{14}$	$+\frac{7}{14}$	$+\frac{6}{12}$	$+\frac{6}{12}$	$+\frac{5}{10}$	$+\frac{5}{10}$	$+\frac{4}{8}$	$+\frac{4}{8}$
Cc	$-\frac{64}{16}$	$-\frac{64}{16}$	$-\frac{50.5}{14}$	$-\frac{50.5}{14}$	$-\frac{36}{12}$	$-\frac{36}{12}$	$-\frac{26.5}{10}$	$-\frac{26.5}{10}$	$-\frac{16}{8}$	$-\frac{16}{8}$
cD	$+\frac{40}{16}$	$+\frac{41.5}{16}$ $-\frac{1.5}{16}$	$+\frac{29.5}{14}$	$+\frac{31}{14}$ $-\frac{1.5}{14}$	$+\frac{18}{12}$	$+\frac{19.5}{12}$ $-\frac{1.5}{12}$	$+\frac{11.5}{10}$	$+\frac{13}{10}$ $-\frac{1.5}{10}$	$+\frac{4}{8}$	$+\frac{5.5}{8}$ $-\frac{1.5}{8}$

TABLE XII—*Continued*

Members	16 Panels.		14 Panels.		12 Panels.		10 Panels.		8 Panels.	
	DL	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*	*DL*	*LL*
DE	$+\frac{32}{16}$	$+\frac{35}{16}$ $-\frac{3}{16}$	$+\frac{22.5}{14}$	$+\frac{25.5}{14}$ $-\frac{3}{14}$	$+\frac{12}{12}$	$+\frac{15}{12}$ $-\frac{3}{12}$	$+\frac{6.5}{10}$	$+\frac{9.5}{10}$ $-\frac{3}{10}$	0	$+\frac{3}{8}$ $-\frac{3}{8}$
EF	$-\frac{32}{16}$	$-\frac{35}{16}$ $+\frac{3}{16}$	$-\frac{22.5}{14}$	$-\frac{25.5}{14}$ $+\frac{3}{14}$	$-\frac{12}{12}$	$-\frac{15}{12}$ $+\frac{3}{12}$	$-\frac{6.5}{10}$	$-\frac{9.5}{10}$ $+\frac{3}{10}$		
Fg	$-\frac{24}{16}$	$-\frac{29.5}{16}$ $+\frac{5.5}{16}$	$-\frac{15.5}{14}$	$-\frac{21}{14}$ $+\frac{5.5}{14}$	$-\frac{6}{12}$	$-\frac{11.5}{12}$ $+\frac{5.5}{12}$				
gH	$+\frac{8}{16}$	$+\frac{19.5}{16}$ $-\frac{11.5}{16}$	$+\frac{1.5}{14}$	$+\frac{13}{14}$ $-\frac{11.5}{14}$						
HI	0	$+\frac{15}{16}$ $-\frac{15}{16}$								
abc	$-\frac{120}{16}$	$-\frac{120}{16}$	$-\frac{91}{14}$	$-\frac{91}{14}$	$-\frac{66}{12}$	$-\frac{66}{12}$	$-\frac{45}{10}$	$-\frac{45}{10}$	$-\frac{28}{8}$	$-\frac{28}{8}$
cde	$-\frac{152}{16}$	$-\frac{152}{16}$	$-\frac{113.5}{14}$	$-\frac{113.5}{14}$	$-\frac{78}{12}$	$-\frac{78}{12}$	$-\frac{51.5}{10}$	$-\frac{51.5}{10}$	$-\frac{28}{8}$	$-\frac{28}{8}$
efg	$-\frac{216}{16}$	$-\frac{216}{16}$	$-\frac{152.5}{14}$	$-\frac{152.5}{14}$	$-\frac{102}{12}$	$-\frac{102}{12}$	$-\frac{58.5}{10}$	$-\frac{58.5}{10}$		
ghi	$-\frac{248}{16}$	$-\frac{248}{16}$	$-\frac{169.5}{14}$	$-\frac{169.5}{14}$						
CE	$+\frac{160}{16}$	$+\frac{160}{16}$	$+\frac{117.5}{14}$	$+\frac{117.5}{14}$	$+\frac{84}{12}$	$+\frac{84}{12}$	$+\frac{53.5}{10}$	$+\frac{53.5}{10}$	$+\frac{32}{8}$	$+\frac{32}{8}$
EG	$+\frac{224}{16}$	$+\frac{224}{16}$	$+\frac{162.5}{14}$	$+\frac{162.5}{14}$	$+\frac{108}{12}$	$+\frac{108}{12}$	$+\frac{66.5}{10}$	$+\frac{66.5}{10}$		
GI	$+\frac{256}{16}$	$+\frac{256}{16}$	$+\frac{173.5}{14}$	$+\frac{173.5}{14}$						

Stresses in diagonal members = panel load $\times$ coefficient $\times \frac{\text{length } aC}{\text{depth } Cc}$.
Stresses in vertical members = panel load $\times$ coefficient $\times$ 1.
Stresses in chord members = panel load $\times$ coefficient $\times \frac{\text{length } ac}{\text{depth } Cc}$.

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